MATLAB implementation of memristor based Chua’s circuit and its chaos control

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Abstract—Recently, there has been an extensive interest in research on the analysis and control of chaotic behaviour in nonlinear systems. Various control problems can be defined for such systems, such as targeting the trajectories to a desired point, stabilizing unstable periodic orbits etc. In this paper, necessity of Chaos control in Memristor based circuit is discussed. A MATLAB Simulink model is presented for a Memristor based Chua’s circuit & feedback control method is implemented for the purpose of Chaos control in the same model.

Index Terms— Chaos, memristor, chaotic behavior, chaos control, feedback control method.

I. INTRODUCTION

Over the past two decades, chaos control has become an important criterion in academic research and practical applications. If Chaos is not controlled, it may result in disaster and collapse of the dynamical system.

Generally speaking, there are two ways to control chaos: feedback control [1-3] and non-feedback control [4, 5]. The non-feedback control is mainly suitable for occasions that require high speed control such as fast electro-optical system. However, due to the weak robustness and the computational arduousness that a non-feedback control may encounter, the feedback control is preferred in some applications.

In this paper, the feedback control method for controlling chaos in a memristor based Chua’s circuit has been implemented. The memristor i.e. memory resistor was postulated as the fourth circuit element by Leon.O.Chua in 1971 [6]. In May 2008, thirty-seven years after Leon Chua’s proposal, the memristor in device form was developed by Stanley Williams and his group in the information and quantum system (IQS) lab at HP [7].

In upcoming days, memristor may become more useful and beneficial in electronic engineering, because this is the only fundamental circuit element which gives the relation between the charge q that flows through a circuit and the flux \( \phi \) in the circuit i.e. \( dq = Mdq \) [8]. It can remember how much current has gone through it and saves its electronic state when power is switched off. By remembering its electrical state it could replace RAM in the future. Memristor will be cheaper, faster and would be able to hold more memory density than flash memory, thus making computers more affordable, smaller and more powerful. Researchers believe that the memristors could be replaced DRAMs by 2014 and may be Hard Disk by 2016 [7]. But,
since memristor has nonlinear characteristics so it may exhibit chaotic dynamics. So, Chaos control in memristor based circuits is very essential.

II. STUDY OF CHAOTIC BEHAVIOUR IN MEMRISTOR BASED CIRCUIT USING MATLAB

The memristor-based circuit, used in this paper has been derived from Chua’s oscillator by replacing Chua’s diode with an active device (within a dotted rectangular box in figure 1) made up of the parallel between a negative resistance and a memristor with the following piece-wise linear (PWL) monotone-increasing odd symmetric charge $q$-flux $\varphi$ characteristic [8].

$$q(\varphi) = b\varphi + \frac{a-b}{2}(|\varphi + 1| - |\varphi - 1|)$$  \hspace{1cm} (1)

Where coefficients $a$ and $b$ are positive since the device is passive.

This passivity implies the necessity to add a suitably-negative resistance of magnitude $R_2$ in parallel with the memristor so as to obtain a locally-active device, i.e. a device characterized by a $q - \varphi$ non-linearity with a negative slope either on the central segment or on the two outer segments. This is an essential requirement for the occurrence of chaotic behaviour in an autonomous dynamical system [8].

The circuit equations of the oscillator of figure 1 are:

$$C_1 \frac{dv_1}{dt} = \frac{v_2 - v_3}{R_1} + \frac{v_1}{R_1} - i$$

$$C_2 \frac{dv_2}{dt} = -l \frac{v_2 - v_1}{R_1}$$

$$L \frac{di}{dt} = v_2 - ri$$

$$\frac{d\varphi}{dt} = v_2$$  \hspace{1cm} (2)

Where the significance of each symbol is shown in figure 1 the memductance of the memristor is defined as [8]

$$W(\varphi) = \frac{dq(\varphi)}{d\varphi}$$

Using the chain rule, memristor current $i$ may be expressed as:

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\[
\frac{di}{dt} = \frac{dq}{d\varphi} \frac{d\varphi}{dt} = W(\varphi)v_1
\]

Charge differentiation of equation “(1),” yields the following expression for \( W(\varphi) \):

\[
W(\varphi) = b + \frac{a-b}{2}(\text{sgn}(\varphi + 1) - \text{sgn}(\varphi - 1)).
\]

Defining state variables are: \( x_1 = v_1, x_2 = v_2, x_3 = -R\bar{t}, \) and \( x_4 = \varphi(R_1C_2)^{-1} \) (each state variable has the dimensions of a voltage), letting \( \alpha = C_2(C_1)^{-1}, \beta = R_1^2C_2(L)^{-1}, \gamma = R_1rC_2(L)^{-1} \) and \( \xi = R_1(R_2)^{-1} \) and considering normalized time variable \( \tau = t(\bar{t})^{-1} \), where \( \bar{t} = R_1C_2 \) is the system time scale, equation “(2),” may be rewritten as:

\[
\begin{align*}
\frac{dx_1}{d\tau} &= \alpha[x_1 + (\xi - 1)x_1 - \bar{W}(x_4)x_1] \\
\frac{dx_2}{d\tau} &= x_3 - x_2 + x_1 \\
\frac{dx_3}{d\tau} &= -\beta x_2 - \gamma x_3 \\
\frac{dx_4}{d\tau} &= x_1
\end{align*}
\]

Where

\[
\bar{W}(x_4) = R_1W(x_4)
\]

\[
= b + \frac{\bar{a} - \bar{b}}{2}(\text{sgn}(\varphi + 1) - \text{sgn}(\varphi - 1))
\]

With

\[
\bar{a} = R_1a \quad (6) \]

\[
\bar{b} = R_1b \quad (7)
\]

Choosing the Initial conditions as \([0.1, -0.1, 0.1, 0]\), setting circuit element values to \( R_1 = 1.5297 \, k\Omega, \) \( R_2 = 1 \, k\Omega, \) \( r = 41.185 \, \Omega, \) \( C_1 = 10nF, \) \( C_2 = 100nF \) and \( L = 18mH \), taking system parameters numerically as \( \alpha = 10, \beta = 13, \gamma = 0.35, \xi = 1.5, \bar{a} = 0.3, \bar{b} = 0.8 \) [8]. Equations “(4),” and “(5),” are simulated in MATLAB/SIMULINK, using the model as shown in figure 2. Details of the subsystem used have been given in figure 3. It has been observed that this system exhibits chaotic behaviour as shown in figure 4.
Figure 2: MATLAB/SIMULINK model of memristor based chaotic circuit

Figure 3: Details of subsystem used in figure 2
The chaotic attractors projected on state space $x_1$-$x_2$-$x_4$ and $x_3$-$x_4$-$x_2$ have been shown in figure 5 & 6 respectively.

III. CONTROLLING CHAOS IN MEMRISTOR BASED CIRCUIT

For controlling chaos in memristor based model used in this paper, a linear switched feedback controller has been designed by solving a set of LMIs based on a common lyapunov function [9]. For this purpose, firstly a control input is added into the first state of memristor based Chua’s circuit, so that the controlled system becomes:

$$\frac{dx_1}{dt} = a(x_1 + (\xi - 1)x_1 - \bar{W}(x_4)x_1) + u,$$

$$\frac{dx_2}{dt} = x_3 - x_2 + x_1,$$
Above system can be represented in the form of a piecewise linear system, defined by equation “(9),”

\[
\dot{x} = \begin{cases} 
A_1 x + B u_1 |x_4| < 1 \\
A_2 x + B u_2 |x_4| > 1 
\end{cases} 
\]  

(9)

Where

\[
x = [v_1, v_2, \xi, \eta]^T \quad \text{And} \quad B = [1 \ 0 \ 0 \ 0]^T
\]

\[
A_1 = \begin{bmatrix}
\alpha [\xi - 1] - \tilde{a} & \alpha & 0 & 0 \\
1 & -1 & 1 & 0 \\
0 & -\beta & -\gamma & 0 \\
1 & 0 & 0 & 0
\end{bmatrix}
\]

\[
A_2 = \begin{bmatrix}
\alpha [\xi - 1] - \tilde{b} & \alpha & 0 & 0 \\
1 & -1 & 1 & 0 \\
0 & -\beta & -\gamma & 0 \\
1 & 0 & 0 & 0
\end{bmatrix}
\]

For chaos control, a linear switched state feedback controller is used which can be represented as

\[
\begin{cases}
 u_1 = K_1 x , \\
 u_2 = K_2 x ,
\end{cases} \quad \begin{cases}
 |x_4| < 1 \\
 |x_4| > 1,
\end{cases} \quad \text{(10)}
\]

Where \(K_1\) and \(K_2\) are the feedback gains, which have to be calculated. For this purpose, a globally quadratic Lyapunov function candidate \(V\) is considered, which is a continuous function & is represented by equation “(11),”

\[
V = x^T P x, \quad \text{(11)}
\]

If a positive matrix \(P = P^T\) is such that, the inequalities “(12),” are true then every trajectory of the state variable used tends to zero exponentially.

\[
\begin{cases}
P A_1 + A_1^T P + P B K_1 + K_1^T B^T P < 0 \\
P A_2 + A_2^T P + P B K_2 + K_2^T B^T P < 0,
\end{cases} \quad \text{(12)}
\]

The condition “(12),” is not still representing linear matrix inequalities and cannot be implemented by using standard numerical software due to the terms \(P B K_i (i \in 1, 2)\). An appropriate state feedback controller gain \(K_i (i \in 1, 2)\) can not be found by using a convex optimization algorithm yet. Pre-and post-multiplying by \(P^{-1}\) respectively, and making the matrix change defined by

\[
P^{-1} = L, \quad E_1 = K_1 P^{-1}, \quad E_2 = K_2 P^{-1}
\]

The inequality “(13),” is obtained
\[
\begin{align*}
\begin{cases}
A1L + LA1^T + BE1 + E1^TB^T < 0 \\
A2L + LA2^T + BK2 + K2^TB^T < 0,
\end{cases}
\end{align*}
\]

By using MATLAB LMI Control Toolbox, LMIs (13), has been solved to get a feasible solution of Lyapunov function & feedback gains $K_1, K_2$ as represented by equations “(14)” &“(15),” respectively.

\[
P = \begin{bmatrix}
4.4379 & 11.3172 - 0.8315 \\
11.3172 & 38.5625 - 2.1909 \\
-0.8315 & -2.1909 + 1.0673 \\
1.8437 & 4.7984 - 0.6943
\end{bmatrix}
\]

\[
K_1 = \begin{bmatrix}
-18.7323 \\
-13.7323
\end{bmatrix}
\]

\[
K_2 = \begin{bmatrix}
-65.654 & 2 \\
-65.6542 & 0.5401 - 7.0532
\end{bmatrix}
\]

Finally, substituting the values of feedback gains $K_1$ and $K_2$ in equation“(10),” the linear switched state feedback controller is obtained. In order to illustrate the effectiveness of the proposed Controller “(10),” a numerical simulation has been carried out in MATLAB/SIMULINK as shown in figure 7.

Figure 7: MATLAB/SIMULINK model of MCC system with linear switched state feedback controller

Responses of the controlled memristor based Chua’s circuit have been shown in figure 8. From this figure, it can be observed that Chaos has been controlled & four state variables have a tendency to reach the equilibrium points [0, 0, 0, 0] in finite time values.
Figure 8(a-d): Controlled response of memristor based Chua’s circuit.
CONCLUSIONS

In this paper, an attempt has been made to explain the method of Chaos control using a linear switched controller in the feedback path. We can observed that, from figure 8(a-d) the results into successful chaos control with reduced settling times (2.5 s for $x_1$, 2 s for $x_2$, 5.1 s for $x_3$, 16 s for $x_4$). Maximum overshoots as well as settling times using this scheme are quite less and this scheme can control chaotic motion to an equilibrium point by numerical simulation. It can also be implemented by using simple components; hence is suitable for the chaos control in MCC used.

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