Abstract- We present a modified structure of 2-D cdf 9/7 wavelet transforms based on adaptive lifting in image coding. Instead of alternately applying horizontal and vertical lifting, as in present practice, Adaptive lifting performs lifting-based prediction in local windows in the direction of high pixel correlation. Hence, it adapts far better to the image orientation features in local windows. The predicting and updating signals of Adaptive lifting can be derived even at the fractional pixel precision level to achieve high resolution, while still maintaining perfect reconstruction. To enhance the performance of adaptive based modified structure of 2-D CDF 9/7 is coupled with SPIHT coding algorithm to improve the drawbacks of wavelet transform. Experimental results show that the proposed modified scheme based image coding technique outperforms JPEG 2000 in both PSNR and visual quality, with the improvement up to 6.0 dB than existing structure on images with rich orientation features.

Keywords: JPEG2000, Compression, Biorthogonal wavelets, CDF9/7, Lifting scheme, SPIHT.

I. INTRODUCTION

Wavelet family is broad. Wavelet basis choice is conditioned by the application at hand or the given objective. In coding, some wavelets are more adequate for smooth regions and others behave better near discontinuities. Hence, many researchers have proposed adaptive schemes that modify the underlying wavelet basis according to local signal characteristics. Filter banks were the fundamental tool to create discrete wavelet transforms. They are formed by the analysis and synthesis low pass and high pass filters and the intermediate stage composed by a down- and up-sampling. Initially, the complexity and challenge of adaptively was to assure the filter bank reversibility in order to recover the original data. The wavelet transforms with spatially adaptive mother wavelets, chosen according to the underlying local signal characteristics are expected to be more effective in representing such signals. The lifting framework for wavelet transforms provides the flexibility of designing wavelet transforms with such nonlinear basis functions. Each one dimensional (1-D) Discrete Wavelet Transform (DWT) can be factored in to one or more lifting stages. In JPEG2000, the Cohen–Daubechies–Feauveau (CDF) 9/7-tap wavelet filter was implemented for lossy image compression by using the lifting scheme. On the basis of the CDF 9/7-tap wavelet filter using the lifting scheme, scientists have proposed several other wavelet filters that have different coefficients. In the designing process, they use a parameter a to find these coefficients. Several 9/7-tap wavelet filters have different coefficients. Daubechies and Sweldens (1998) used the lifting scheme and the factoring method with the irrational number a 5 1.25 to generate a filter (for the CDF 9/7-tap wavelet filter, t 5 1.230174. . .). This filter performs at the same level but its parameter t is less complex than that of the CDF 9/7-tap wavelet filter. Although the wavelet filters proposed by Guangjun et al. (2001) and Liang et al. (2003) perform equally well or slightly better than the CDF 9/7-tap wavelet filter, their performances are not optimal.

The main concern is whether there is any other a value and its corresponding coefficients of wavelet filters that can provide better performance with less complex coefficients than those of the CDF 9/7-tap wavelet filter. In this paper, the proposed structure is realized by introducing an adaptive filtering framework into a 2D direct lifting modified structure based on 9/7 DWT. The lifting modified structure of DWT has been proposed for the integer-to-integer transforms [10]. For 2D signals such as images, a 2D adaptive lifting structure of DWT is realized by applying the 1D lifting structures to images twice, vertically and horizontally. Iwahashi et O have proposed the 2D direct lifting structure based on 9/7 DWT, by interchanging and merging some lifting in the 2D separable lifting structure of 9/7 DWT [11]. Our proposed modified structure of 9/7 DWT can design 2D adaptive lifting structures followed by it is compressed using Set Partitioning in Hierarchical Trees (SPIHT) algorithm, the most popularly used image compression algorithm (Said, William and Pearlman 1996). The proposed adaptive filtering is realized by changing the sampling matrix by sub-regions of images, according to feature directions of the sub-regions. With advantages of the 2D adaptive structure and the adaptive filtering, the proposed structure improves the performance of the lossy image coding application. Finally, lossy im-age coding results of the proposed structure is compared with previous work done by P.Getreuer[2] and Wang Tianhu[3] are shown to...
validate the advantage of the proposed structure. Section 2 summarizes wavelet transform. We propose CDF 9/7 Wavelet and adaptive lifting in Sect. 3. SPIHT coding scheme and Compression Quality Evaluation in Sect. 4. Section 5 summarizes Results and Discussion and conclusions are presented in Sect. 6.

II. WAVELET TRANSFORMS

The wavelet transform (WT), in general, produces floating point coefficients. Although these coefficients are used to reconstruct an original image perfectly in theory, the use of finite precision arithmetic and quantization results in a lossy scheme. Recently, reversible integer WT’s (WT’s that transform integers to integers and allow perfect reconstruction of the original signal) have been introduced [3, 4]. In [5], Claypoole introduced how to use the lifting scheme presented in [6], where Sweldens [6] showed that the convolution based biorthogonal WT can be implemented in a lifting-based scheme as shown in Fig. 1 for reducing the computational complexity. Note that only the decomposition part of WT is depicted in Fig. 1 because the reconstruction process is just the reverse version of the one in Fig. 1. The lifting-based WT consists of splitting, lifting, and scaling modules and the WT is treated as prediction error decomposition. It provides a complete spatial interpretation of WT. In Fig. 1, let X(n) denote the input signal, and X(e) and X(o) be the decomposed output signals, where they are obtained through the following three modules of lifting-based 1DWT:

Splitting: In this module, the original signal X is divided into two disjoint parts, i.e., that denote all even-indexed and odd-indexed samples of X, respectively [8].

Lifting: In this module, the prediction operation P is used to estimate X(e)(n) from X(n) and results in an error signal d(n) which represents the detailed part of the original signal. Then we update d(n) by applying it to the update operation U, and the resulting signal is combined with X(e) to s(n) estimate, which represents the smooth part of the original signal.

Scaling: A normalization factor is applied to d(n) and s(n), respectively. In the even-indexed part s(n) is multiplied by a normalization factor K to produce the wavelet subband X(e). Similarly, in the odd-indexed part the error signal d(n) is multiplied by K, to obtain the wavelet subband X(o). Note that the output results of X(e) and X(o) obtained by using the lifting based WT are the same as those of using the convolution approach for the same input even if they have completely different functional structures. Compared with the traditional convolution-based WT, the lifting-based scheme has several advantages. First, it makes optimal use of similarities between the highpass and lowpass filters; the computation complexity can be reduced by a factor of two. Second, it allows a full inplace calculation of the wavelet transform. In other words, no auxiliary memory is needed.

III. ADAPTIVE LIFTING BASED CDF 9/7

This section deals with biorthogonal wavelet 9/7. The lifting scheme of the biorthogonal transform 9/7 goes through four steps: two prediction operators and two update operators as shown in Fig. 2 [11, 13].

Since the lifting scheme does not use Fourier analysis to compute the DWT, it can be used in situations where translation and dilation is impossible. One example would be near boundaries of a finite signal where normal Fourier techniques would provide border distortion or artifacts.

A. LIFTING STEP EXTRACTION: As mentioned above the lifting scheme is an alternative technique for performing the DWT using biorthogonal wavelets. In order to perform the DWT using the lifting scheme the corresponding lifting and scaling steps must be derived from the biorthogonal wavelets. The analysis filters of the particular wavelet are first written in polyphase matrix form shown below.

\[
P(z) = \begin{bmatrix}
    h_{even}(z) & g_{even}(z) \\
    h_{odd}(z) & g_{odd}(z)
\end{bmatrix}
\]

(1)

The poly phase matrix is a 2 x 2 matrix containing the analysis low-pass and high-pass filters each split up into their even and odd polynomial coefficients and normalized. From here the matrix is factored into a series of 2 x 2 upper and lower triangular matrices each with diagonal entries equal to 1. The upper triangular matrices contain the coefficients for the predict steps and the lower triangular matrices contain the coefficients for the update steps. A matrix consisting of all O’s with the exception of the diagonal values may be extracted to derive the scaling step coefficients. The polyphase matrix is factored into the form shown in the equation below, a is the coefficient for the predict step and p is the coefficient for the update step.

\[
P(z) = \begin{bmatrix}
1 & a(1+z^{-1}) \\
0 & 1
\end{bmatrix} \begin{bmatrix}
1 & 0 \\
1 & 1
\end{bmatrix}
\]

(2)

An example of a more complicated extraction having multiple...
predict and update steps as well as scaling steps is shown below; a is the coefficient for the first predict step, p is the coefficient for the first update step, A, is the coefficient for the second predict step, 5 is the coefficient for the second update step, \(k_1\) is the odd sample scaling coefficient, and \(k_2\) is the even sample scaling coefficient.

\[
P(z) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a(1 + z^{-1}) & 0 \\ 0 & c(1 + z^{-1}) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ d(1 + z) & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ 0 \end{bmatrix} \begin{bmatrix} k_2 \\ 1 \end{bmatrix}
\]

(3)

According to matrix theory, any matrix having polynomial entries and a determinant of 1 can be factored as described above. Therefore every FTR wavelet or filter bank can be decomposed into a series of lifting and scaling steps. Daubechies and Sweldens discuss lifting step extraction in further detail. [6] The lifting step extraction for the CDF9/7 biorthogonal wavelets is shown below.

**Table I. Adaptive Lifting Parameters Of Modified Structure Of CDF 9/7**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>-1.5861343420693648</td>
</tr>
<tr>
<td>b</td>
<td>-0.0529801185718856</td>
</tr>
<tr>
<td>c</td>
<td>0.8829110755411875</td>
</tr>
<tr>
<td>d</td>
<td>0.4435068520511142</td>
</tr>
<tr>
<td>(k_1), (k_2)</td>
<td>0.230174104914126 and 1.62578613232319229</td>
</tr>
</tbody>
</table>

A total of four lift steps are required, two predict and two update steps, to perform the proposed modified structure of CDF 9/7 DWT by adapting new scaling values of scaling coefficients of \(k_1\) and \(k_2\). The coefficients for predict and update steps are enlisted in table I. The predict and update equations for the CDF 9/7 filter are shown below.

**Predict 1:**

\[
\text{odd}_{\text{new}} = \text{odd}_{\text{old}} + [a(\text{even}_{\text{left}} + \text{even}_{\text{right}})]
\]

**Update 1:**

\[
\text{even}_{\text{new}} = \text{even}_{\text{old}} + [b(\text{odd}_{\text{left}} + \text{odd}_{\text{right}})]
\]

**Predict 2:**

\[
\text{odd}_{\text{new}} = \text{odd}_{\text{old}} + [c(\text{even}_{\text{left}} + \text{even}_{\text{right}})]
\]

**Update 2:**

\[
\text{even}_{\text{new}} = \text{even}_{\text{old}} + [d(\text{odd}_{\text{left}} + \text{odd}_{\text{right}})]
\]

**Scale odd:**

\[
\text{odd}_{\text{new}} = [k_1 \times \text{odd}_{\text{old}}]
\]

**Scale even:**

\[
\text{even}_{\text{new}} = [k_2 \times \text{even}_{\text{old}}]
\]

The floor function is used for all the predict, update and scale equations to provide an integer-to integer transform. The forward CDF 9/7 DWT using the lifting scheme is shown in Figure 3. The CDF 9/7 DWT consists of four lifting steps and two scaling steps. The first lifting step (predict step 1) is applied to the original row of samples and the results then safely overwrite the odd samples in the original signal for use in the next lifting step.

\[
\text{odd}_{\text{new}} = \text{odd}_{\text{old}} + [a(\text{even}_{\text{left}} + \text{even}_{\text{right}})]
\]

(4)

The second lifting step (update step 1) is applied to the results from the first lifting step and the remaining even samples of the original signal. The results then safely overwrite the even samples in the signal

\[
\text{even}_{\text{new}} = \text{even}_{\text{old}} + [b(\text{odd}_{\text{left}} + \text{odd}_{\text{right}})]
\]

and \(a=-1.5861343420693648\)

**Row of samples:**

\[4 \quad 7 \quad 3 \quad 5 \quad 9 \quad 6\]

Lifting step 1 results: \[-5 \quad -15 \quad -23\]

New row of samples: \[4 \quad 5 \quad -15 \quad 9 \quad -23\]

**The second lifting step (update step 1) is applied to the results from the first lifting step and the remaining even samples of the original signal. The results then safely overwrite the even samples in the signal**

\[
\text{odd}_{\text{new}} = \text{odd}_{\text{old}} + [a(\text{even}_{\text{left}} + \text{even}_{\text{right}})]
\]

and \(a=-1.5861343420693648\)

**Row of samples:**

\[4 \quad 5 \quad 3 \quad 9 \quad 6\]

Lifting step 2 results: \[4 \quad 4 \quad 11\]

New row of samples: \[4 \quad 5 \quad -15 \quad 11 \quad -23\]

The third lifting step (predict step 2) is applied to the results from the first and second lifting steps. The results then safely overwrite the results from the first lifting step.

\[
\text{odd}_{\text{new}} = \text{odd}_{\text{old}} + [a(\text{even}_{\text{left}} + \text{even}_{\text{right}})]
\]

and \(a=-0.0529801185718856\)

**Row of samples:**

\[4 \quad 5 \quad 3 \quad 9 \quad 6\]

Lifting step 3 results: \[2 \quad -2 \quad -4\]

New row of samples: \[4 \quad 2 \quad 4 \quad -2 \quad 11 \quad -4\]

The fourth lifting step (update step 2) is applied to the results from the second and third lifting steps. The results then safely overwrite the results from the second lifting step.

\[
\text{even}_{\text{new}} = \text{even}_{\text{old}} + [b(\text{odd}_{\text{left}} + \text{odd}_{\text{right}})]
\]

and \(C=0.8829110755411875\)

**Row of samples:**

\[4 \quad 5 \quad 4 \quad -15 \quad 11 \quad -23\]

Lifting step 4 results: \[5 \quad 4 \quad 8\]

New row of samples: \[5 \quad 2 \quad 4 \quad -2 \quad 8 \quad -4\]

The first scaling step is applied to the results from the third lifting step. The results then safely overwrite the results from the third lifting step. The results from this step are the wavelet coefficients.

\[
\text{odd}_{\text{new}} = [k_1 \times \text{odd}_{\text{old}}]
\]

and \(k_1=0.230174104914126\)

**Row of samples:**

\[5 \quad 2 \quad 4 \quad -2 \quad 8 \quad -4\]

Lifting step 4 results: \[2 \quad -3 \quad -5\]

New row of samples: \[5 \quad 2 \quad 4 \quad -3 \quad 8 \quad -5\]

Wavelet coefficients: \[2 \quad -3 \quad -5\]

The second scaling step is applied to the results from the fourth lifting step. The results then safely overwrite the results from the fourth lifting step. The results from this step are the scaling coefficients.

\[
\text{even}_{\text{new}} = [k_2 \times \text{even}_{\text{old}}]
\]

and \(k_2=1.62578613232319229\)

**Row of samples:**

\[5 \quad 2 \quad 4 \quad -3 \quad 8 \quad -5\]

Lifting step 4 results: \[4 \quad 3 \quad 6\]
New row of samples: 4 2 3 -3 6
Scaling coefficients: 4 3 6

IV. SPIHT Coding Scheme

When the decomposition/forward lifting image is obtained, we try to find a way to code the wavelet coefficients into an efficient result, taking redundancy and storage space into consideration. SPIHT [7] is one of the most advanced schemes available, even outperforming the state-of-the-art JPEG 2000 in some situations. The basic principle is the same; a progressive coding is applied, processing the image respectively to a lowering threshold. The difference is in the concept of zero trees (spatial orientation trees in SPIHT). This is an idea that takes into consideration bounds between coefficients across subbands at different levels [9]. The first idea is always the same: if there is a coefficient at the highest level of the transform in a particular subband which considered insignificant against a particular threshold, it is very probable that its descendants in lower levels will be insignificant too.

Therefore we can code quite a large group of coefficients with one symbol. Fig. 4 shows how a spatial orientation tree is defined in a pyramid constructed with recursive four subbands splitting. The coefficients are ordered hierarchically. According to this relationship, the SPIHT algorithm saves many bits that specify insignificant coefficients [15]. The algorithm of SPIHT is discussed as a First step, the original image is decomposed into ten subbands. Then the method finds the maximum and the iteration number. Second, the method puts the DWT coefficients into a sorting pass that finds the significance coefficients in all coefficients and encodes the sign of these significance coefficients.

![Fig. 4. Parent-child relationship.](image)

Third, the significance coefficients that can be found in the sorting pass are put into the refinement pass that uses two bits to exact the reconstruct value for approaching to real value. The first second and third steps are iterative, and then iteration decreases the threshold \(T_n = T_{n-1}/2\) and the reconstructive value \(R_n = R_{n-1}/2\). As a fourth step, the encoding bits access entropy coding and then transmit. The result is in the form of a bit stream. All of the wavelet-based-image encoding algorithms improve the compression rate and the visual quality.

Compression Quality Evaluation: The Peak Signal to Noise Ratio (PSNR) is the most commonly used as a measure of quality of reconstruction in image compression. The PSNR are identified using the following formulate:

\[
\text{PSNR} = 10 \log_{10} \left( \frac{\text{Dynamic range of image}}{\text{MSE}} \right)
\]

Mean Square Error (MSE) which requires two M × N grayscale images I and \(\hat{I}\), where one of the images is considered as a compression of the other is defined as:

\[
MSE^2 = \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} (I(i,j) - \hat{I}(i,j))^2
\]

Here, an image is encoded on 8 bits. It is represented by 256 gray levels, which vary between 0 and 255, the extent of image dynamics is 255.

V. Results and Discussion

We are interested in lossy compression methods based on 2D wavelet transforms because their properties are interesting. Indeed, the 2D wavelets transform combines good spatial and frequency locations. We applied the proposed algorithm on test image ‘Barbara’ of size 256 X 256 encoded by 0.25 to 8 bit rates in bpp.

The importance of our work lies in the possibility of reducing the rates for which the image quality remains acceptable. Estimates and judgments of the compressed image quality is given by the PSNR evaluation parameters shown below in figure 5 illustrates the compressed reconstructed image quality with adaptive lifting variations for different bit-rate values (number of bits per pixel) for 2D Barbara Image. To show the performance of the proposed method, we will now make a comparison between our proposed scheme of CDF 9/7 (proposed) coupled with the SPIHT coding, and existing lifting structures of CDF 9/7 Pascal Getreuer[2] and Wang Tianhui[3].

For each application we varied the bit-rate 0.25 to 8 bpp for 1 to 8 levels, and we calculate the PSNR. The results obtained are given in Table II. The comparison in terms of image quality for the three algorithms is given in table II. By comparing the different values of PSNR, we shows clearly the effectiveness of our proposed modified structure of algorithm in terms of compressed image quality which is 6 db better than existing Wang Tianhui[3] for level 8 and close to Pascal Getreuer[2] from level 4 to 8.

CONCLUSION

The proposed modified structure of CDF 9/7 transform based on adaptive lifting has been applied to 256X256 8 bit images. This adaptive lifting transform appears promising for image compression. It reduces edge artifacts and ringing and gives improved PSNR for edge dominated images like Barbara. For smooth images like Barbara, 9/7 transform gives much better performance of 6 db than previous work [3] for level 8 at different bit rates and closer to work done[2] for all levels. Further, all this can be achieved without extra cost on coding the filter decisions.
TABLE II. PSNR OF ADAPTIVE LIFTING BASED 9/7 CDF AT 2 TO 6 LEVELS WITH DIFFERENT RATES

<table>
<thead>
<tr>
<th>Rate</th>
<th>level</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>proposed</td>
<td>11.47</td>
<td>13.28</td>
<td>13.28</td>
<td>22.36</td>
</tr>
<tr>
<td>0.5</td>
<td>Getreuer [2]</td>
<td>15.18</td>
<td>16.24</td>
<td>16.24</td>
<td>26.09</td>
</tr>
<tr>
<td></td>
<td>proposed</td>
<td>11.47</td>
<td>13.28</td>
<td>13.28</td>
<td>22.36</td>
</tr>
<tr>
<td></td>
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<td>13.28</td>
<td>13.28</td>
<td>22.36</td>
</tr>
</tbody>
</table>

Fig. 5. Shows the effect of adaptive lifting at 1 to 8 lifting levels with 0.25 to 6 bit rate in bpp for Barbara image.

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