Adaptive Control of Saturated Induction Motor with Uncertain Load Torque

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Abstract—The most popular method for high performance control of induction motor is field oriented control. Variation of load torque cause input-output coupling, steady state tracking errors and deteriorated transient response in this control. A nonlinear adaptive controller is designed for induction motor with magnetic saturation which includes both electrical and mechanical dynamics. The control algorithm contains a nonlinear identification scheme which asymptotically tracks the true value of the unknown time varying load torque. Once this parameter is identified, the two control goals of regulating rotor speed and rotor flux amplitude are decoupled, so that power efficiency can be improved without affecting speed regulation.

Index Terms—Adaptive control, Induction motor with saturation, Matlab/Simulink

I. INTRODUCTION

Induction motor is known for constant speed applications. It is maintenance free, simple in operation, rugged and generally less expensive than either DC or synchronous motors. On the other hand its model is more complicated than other machines and because of this, it is considered as ‘the benchmark problem in nonlinear control’. There are many approaches to induction motor control. Field oriented control of induction motor was developed by Blaschke. Field oriented control achieves input-output decoupling. Applications clearly indicate that even though nonlinearities may be exactly modeled, the physical parameters involved are most often not precisely known. This motivated further studies on adaptive versions of feedback linearization and input-output linearization [1], since cancellations of nonlinearities containing parameters are required. Load torque and rotor resistance are estimated. The common assumption made in these control laws is the linearity of the magnetic circuit of the machine. In many variable torque applications, it is desirable to operate the machine in the magnetic saturation region to allow the machine to develop higher torque. The paper is organized as follows. We shall first describe a fifth model [2] of induction motor and then field oriented model. An adaptive control of induction motor with saturation is developed which estimates a time varying load torque.

II. INDUCTION MOTOR MODEL

A. Model of Induction Motor

An induction motor is made by three stator windings and three rotor windings.

\[
\begin{align*}
R_s i_{sb} + \frac{d\psi_{sb}}{dt} &= u_{sb} \\
R_r i_{rd} + \frac{d\psi_{rd}}{dt} &= 0 \\
R_s i_{sa} + \frac{d\psi_{sa}}{dt} &= u_{sa} \\
R_r i_{rq} + \frac{d\psi_{rq}}{dt} &= 0
\end{align*}
\]  

(1)

Where R, i, \psi, u denote resistance, current, flux linkage, and stator voltage input to the machine; the subscripts s and r stand for stator and rotor, (a,b) denote the components of a vector with respect to a fixed stator reference frame, (d',q') denote the components of a vector with respect to a frame rotating We now transform the vectors (i_{rd}, i_{rq}), (\psi_{rd}, \psi_{rq}) in the rotating frame (d',q') into vectors (i_{ra}, i_{rb}, \psi_{ra}, \psi_{rb}) in the stationary frame (a,b) by

\[
\begin{align*}
\begin{bmatrix}
i_{ra} \\
i_{rb} \\
\psi_{ra} \\
\psi_{rb}
\end{bmatrix} &=
\begin{bmatrix}
\cos\delta & -\sin\delta \\
\sin\delta & \cos\delta \\
\cos\delta & -\sin\delta \\
\sin\delta & \cos\delta
\end{bmatrix}
\begin{bmatrix}
i_{rd} \\
i_{rq} \\
\psi_{rd} \\
\psi_{rq}
\end{bmatrix}
\end{align*}
\]  

(2)

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The system (1) becomes
\[
\frac{d\psi}{dt} = \frac{R_r}{L_r} \psi + \frac{R_r}{L_r} M_i - n_p \omega \psi_b
\]
\[
\frac{d\psi}{dt} = -\frac{R_r}{L_r} \psi_b + \frac{R_r}{L_r} M_i + n_p \omega \psi_a
\]
\[
\frac{di_a}{dt} = \frac{MR_r}{\sigma L_r^2} \psi + \frac{n_p M}{\sigma L_r^2} \psi_b
\]
\[
-\left(\frac{M^2 R_r + L_r^2 R_s}{\sigma L_r^2}\right) i_a + \frac{1}{\sigma L_s} u_a
\]
\[
\frac{di_b}{dt} = \frac{MR_r}{\sigma L_r^2} \psi_b - \frac{n_p M}{\sigma L_r^2} \psi_a
\]
\[
-\left(\frac{M^2 R_r + L_r^2 R_s}{\sigma L_r^2}\right) i_b + \frac{1}{\sigma L_s} u_b
\] (4)

Where \(i, \psi, u\) denote current, flux linkage and stator voltage input to the machine; the subscripts \(s\) and \(r\) stand for stator and rotor; \((a,b)\) denote the components of a vector with respect to a fixed stator reference frame and \(\sigma = 1 - (M^2/L_s L_r)\).

Let \(u = (u_a u_b)^T\) be the control vector. Let \(\alpha = R_r N/L_r\), \(\beta = (M/\sigma L_r)\), \(\gamma = (M^2 R_r N/\sigma L_r) + (R_s/\sigma L_s)\), \(\mu = (n_M M/L_r)\).

System in (4) can be written in compact form as
\[
\dot{x} = f(x) + u_a g_a + u_b g_b + p_1 f_1 + p_2 f_2 (x)
\] (5)

B. Field Oriented Model

It involves the transformation of vectors \((i_a, i_b)\), \((\psi_a, \psi_b)\) in the fixed stator frame \((a, b)\) into vectors in a frame \((d, q)\) which rotate along with the flux vector \((\psi_a, \psi_b)\); defining
\[\rho = \arctan(\Psi_b/\Psi_a)\]
the transformations are
\[
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix} =
\begin{bmatrix}
\cos \rho & \sin \rho \\
-\sin \rho & \cos \rho
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b
\end{bmatrix}
\]
\[
\begin{bmatrix}
\psi_d \\
\psi_q
\end{bmatrix} =
\begin{bmatrix}
\cos \rho & \sin \rho \\
-\sin \rho & \cos \rho
\end{bmatrix}
\begin{bmatrix}
\psi_a \\
\psi_b
\end{bmatrix}
\] (7)

Since \(\cos \rho = (\psi_a/|\psi|)\), \(\sin \rho = (\psi_b/|\psi|)\), with \(|\psi| = \psi_a^2 + \psi_b^2\).

We now interpret field oriented model as state feedback transformation (involving space change of coordinates and nonlinear state feedback) into a control system of simpler structure. Defining the state space change of coordinates
\[
\omega = \omega
\]
\[
\psi_d = \sqrt{\psi_a^2 + \psi_b^2}
\]
\[
\rho = \arctan(\psi_b/\psi_a)
\]
\[
i_d = \frac{\psi_a i_a + \psi_b i_b}{|\psi|}
\]
\[
i_q = \frac{\psi_a i_b - \psi_b i_a}{|\psi|}
\]
\[
\psi_d = \sqrt{\psi_a^2 + \psi_b^2} = |\psi|
\]
\[
\psi_d = 0.
\] (8)

Substituting the above equations (10) in (9) the system yields the field oriented model
\[
\frac{d\omega}{dt} = \mu \psi_d i_q - \frac{T_i}{J}
\]
\[
\frac{d\psi_d}{dt} = \alpha \beta M i_d
\]
\[
\frac{di_d}{dt} = -\gamma_i d + \alpha \beta \psi_d + n_p \omega i_q + \alpha M \frac{i_d^2}{\psi_d} + \frac{1}{\sigma L_s} u_d
\]
\[
\frac{di_q}{dt} = -\gamma_i d - \beta n_p \omega \psi_d
\]
\[
- n_p \omega i_d - \alpha M \frac{i_d}{\psi_d} + \frac{1}{\sigma L_s} u_q
\]
\[
\frac{d\rho}{dt} = n_p \omega + \alpha M \frac{i_d}{\psi_d}
\] (10)
III. INDUCTION MOTOR CONTROL

A. Control Strategy

Let \( \omega_r(t) \) and \( \psi_r(t) \) be the smooth bounded reference signals for the output variables to be controlled which are speed \( \omega \) and flux modulus \( \psi \). The control strategy is shown in Figure (1). The goal is to design control inputs \( u_d \) and \( u_q \) so that for any unknown time varying \( T_L(t) \), we obtain

\[
\lim_{t \to \infty} [\omega(t) - \omega_r(t)] = 0 \\
\lim_{t \to \infty} [\psi_d(t) - \psi_r(t)] = 0
\]  

(11)

A standard approach for simplifying the dynamics for the design of the speed and flux loops is to use proportional plus integral control loops to generate current command. That is, choose the inputs as

\[
u_d = -k_{d1}(i_d - i_{d\text{ref}}) - k_{d2}\int_{0}^{t}(i_d(\tau) - i_{d\text{ref}}(\tau))d\tau.
\]  

(12)

\[
u_q = -k_{q1}(i_q - i_{q\text{ref}}) - k_{q2}\int_{0}^{t}(i_q(\tau) - i_{q\text{ref}}(\tau))d\tau.
\]  

(13)

Proper choice of the gain results tracking the reference currents so that current dynamics can be ignored. Therefore the system can then be replaced by the following reduced order.

\[
\frac{d\psi_d}{dt} = M\alpha({f^{-1}(\psi_d)} + i_{d\text{ref}})
\]  

(14)

Now \( i_{q\text{ref}} \) and \( i_{d\text{ref}} \) are the inputs and these inputs are used to control \( \omega \) and \( \psi \). Different models are available to incorporate flux saturation. Let’s use the model presented in [3] as given below.

\[
\frac{d\psi_d}{dt} = M\alpha(-f^{-1}(\psi_d) + i_{d\text{ref}})
\]  

(15)

Where \( M \) and \( \alpha \) are at their nominal values. In steady state \( \psi_d = f(i_d) \), which represents the saturation curve for induction motor. In the absence of saturation \( \psi_d = M i_d \) and \( f^{-1}(\psi_d) = \psi_d / M \). Therefore (11) reduces to linear case.

Defining the error variables

\[
\hat{\omega} = \omega - \omega_r \\
\hat{\psi} = \psi - \psi_r
\]  

(16)

Using the reduced motor equations and substituting

\[
T_L(t) = c_0 + c_1(t - t_s)
\]

as in [4]. Where \( c_0 \) and \( c_1 \) are constant coefficients of the first order polynomial used to estimate the time varying parameter \( T_L(t) \). We deduce the error equations.

\[
\frac{d\hat{\omega}}{dt} = \mu\psi_d i_q - \frac{c_0(t)}{J} - \frac{c_1(t)}{J}(t - t_s) - \hat{\omega}
\]

\[
\frac{d\hat{\psi}}{dt} = -M\alpha f^{-1}(\psi_d + i_d) - \hat{\psi}
\]  

(17)

Choosing the Lyapunov function

\[
V = \frac{1}{2}(\gamma_1\hat{\omega}^2 + \psi_d^2 + \gamma_2\hat{\psi}_d^2 + \gamma_3\hat{\psi}_d^2)
\]  

(18)
Where $\gamma_1, \gamma_2, \gamma_3$ are positive design parameters chosen by the designer. $\hat{c}_o = c_o - \hat{c}_o, \hat{c}_1 = c_1 - \hat{c}_1$ are the coefficient estimation errors and $\hat{c}_o, \hat{c}_1$ are estimates of the coefficients.

The desired currents inputs are chosen as follows

$$i_{qref} = \frac{1}{\mu \psi_d} [-k_w \delta^2 + \frac{\hat{c}_o}{J} + \frac{\hat{c}_1}{J}(t - t_o) + \dot{\delta}_e]$$

$$i_{dref} = \frac{1}{M \alpha} [M \alpha f^{-1}(\psi_d) + \dot{\psi}_r - k_{vd}]$$ (19)

Where $k_w$ and $k_{vd}$ are positive design parameters.

The adaptation laws are given by

$$\dot{\hat{c}}_o = \frac{1}{\lambda_2} (\frac{-\lambda_1 \delta e}{J})$$

$$\dot{\hat{c}}_1 = \frac{1}{\lambda_3} (\frac{-\lambda_1 \delta e(t - t_o)}{J})$$ (20)

With

$$\dot{V} = -\lambda_1 k_w \delta^2 - k_{vd} \dot{\psi}_d^2$$ (21)

### IV. SIMULATION

The proposed algorithm has been simulated in Matlab/Simulink for a 15 KW motor, with rated torque 70 Nm and rated speed 220 rad/s, whose data are listed in the Appendix. The simulation test involves the following operating sequences: the unloaded motor is required to reach the rated speed and rated value of 1.3 Wb for rotor flux amplitude. At $t = 2s$ a load torque 40Nm, which is unknown to the controller, is applied. At $t = 5s$, the speed is required to reach 300 rad/s, well above the nominal value, and rotor flux amplitude is weakened. The reference signals for flux amplitude and speed consists of tep functions, smoothed by means of second order polynomials. A small time delay at the beginning of the speed reference trajectory is introduced in order to avoid overlapping between flux and speed transients.

Figure 2 & 3 shows the reference speed and flux.

Simulation results for $f^{-1}(\psi_d) = \psi_d / M$ are shown in [9]. Simulation results for $f^{-1}(\psi_d) = \psi_d^3 / M$ (saturation) are shown in Figure 4 & 5. The controller parameters are chosen as $k_w=12000, k_{vd}=1000, k_{q1}=46000, k_{q2}=88000, k_{q1}=196000, k_{q2}=188000$. Figure 4 shows the actual speed and it tracks the reference speed even though load torque changes as a function of time. The flux response tracks the reference flux as shown in Figure 5. The simulation results show the superior behavior of the control algorithm.
CONCLUSION

In this paper we propose a detailed nonlinear model of an induction motor, an adaptive input-output decoupling control which has some advantages over the classical scheme of field oriented control. With a comparable complexity exact decoupling between speed and flux regulation is achieved and critical parameter is identified. The main drawback of the proposed control is the requirement of flux measurements. Additional research should analyze the influence of sampling error, measurement of noise, simplifying modeling assumptions and unmodeled dynamics. The authors can conclude on the topic discussed and proposed. Future enhancement can also be briefed here.

Appendix A

Induction motor Data

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>R_s</td>
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<tr>
<td>R_r</td>
<td>0.15 Ω</td>
</tr>
<tr>
<td>ψ_r</td>
<td>1.3 Wb rated</td>
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<td>220 rad/sec rated</td>
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<tr>
<td>J</td>
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</tr>
<tr>
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</tr>
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<td>T_L</td>
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</tr>
<tr>
<td>T</td>
<td>Rated power 15 Kw</td>
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</table>

REFERENCES