Kinematics Control and Balancing of Articulated Rovers with Active Suspensions

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Abstract

The paper summarizes a systematic method that was developed for kinematics modeling, analysis and balance control of general high mobility wheeled rovers traversing uneven terrain. The method is based on the propagation of position and orientation velocities through various joints and linkages connecting the rover reference frame to the wheel-terrain contact frames. Actuation kinematics and balance control are formulated for rovers traversing uneven terrains. The main thrust of this paper is applying the method to an articulated rover with an active suspension system to illustrate the proposed kinematics modeling and balancing. Results are provided for the rover moving over several terrains.

Keywords Rover kinematics; balance control; high mobility rovers

1. Introduction

Rovers with high mobility mechanisms are capable of traversing rough terrain and adapting their configurations to the changing terrain topology. They are being used increasingly in diverse applications in both terrestrial and planetary explorations. Rovers with active suspension systems are capable of modifying and adjusting their suspension linkages and joints so as to change their center of mass to avoid tipover while traversing rough and inclined terrain [1]-[2].

A study of the kinematics of a particular rover is reported in [3]. In a recent paper [4], we developed a full kinematics model of articulated rovers and provided analysis of such rovers. Reusable kinematics models and algorithms for manipulators and vehicles are reported in [5] within the framework of object-oriented software. The work reported in [6] employs a simple kinematics model and a state observer to estimate rover position/orientation velocities. A Kalman filter approach is proposed in [7], [8] to estimate wheel contact angles for traction control. A method for ensuring the stability of articulated rovers is discussed in [9].

Wheel driving force and tip-over stability are also considered in [10] where a scheme is proposed to utilize the rover center of mass in order to aid in traversing rough terrain.

In this paper we apply the method described in our previous papers [4] and [12] to an articulated rover with active suspension (ARAS) similar to the NASA JPL Sample Return Rover. A balance control is included and augmented with the rover kinematics to ensure that the rover motion is stable and tipover is prevented.

2. Kinematic Modelling

A high mobility wheeled rover is defined as a rover that consists of a main body connected to a set of wheels via a set of linkages and joints that are adjusted so as to enable it traverse the uneven terrain. The joints and linkages change either passively or actively. Active linkages and joints have actuators through which their values can be controlled, whereas passive ones change their values to comply with the terrain topology.

The goal of kinematics modeling is to relate the motion of the rover body to the motions of the wheels, and vice versa. In order to achieve this, we attach a sequence of frames starting at the rover reference frame going to the suspension joints, steering, etc. and finally ending at the wheel-terrain contact frame. Let \( F_{i-1} \) and \( F_i \) be two consecutive frames in the sequence with their axes denoted by \((x_{i-1}, y_{i-1}, z_{i-1})\) and \((x_i, y_i, z_i)\), respectively. Using the Denavit-Hartenberg (DH) parameters we can describe the transformation between these two frames. The four DH parameters are denoted here by \((\alpha_i, a_i, \lambda_i, d_i)\) where \(\alpha_i\) is the angle from \(z_{i-1}\) to \(z_i\) measured about \(x_{i-1}\); \(a_i\) is the distance from \(z_{i-1}\) to \(z_i\) measured along \(x_i\); \(\lambda_i\) is the angle from \(x_{i-1}\) to \(x_i\) measured about \(z_i\) and \(d_i\) is the distance from \(x_{i-1}\) to \(x_i\) measured along \(z_i\). It is noted that the first parameter \((\alpha_i, a_i, \lambda_i, d_i)\) is equivalent to \((\alpha_{i-1}, a_{i-1}, \theta_i, d_i)\) in the conventional DH notation.

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two parameters \((\varepsilon_i, a_i)\) are associated with the link between the two consecutive frames and are constant. On the other hand, either \(\lambda_i\) or \(d_i\) is variable, depending whether it is revolute or prismatic.

In the case of rovers, we are generally interested in the translational velocity vector \(\dot{u}_i = [\dot{x}_i \dot{y}_i \dot{z}_i]^T\) and rotational velocity \(\dot{\phi}_i = [\dot{\alpha}_i \dot{\beta}_i \dot{\gamma}_i]^T\) vector, where \(\alpha_i\) is the pitch, \(\beta_i\) is the roll and \(\gamma_i\) is the yaw. The \(3 \times 1\) translational velocity vector \(\dot{u}_i\) of the current frame \(F_i\) is dependent on the translational and rotational velocities of the previous frame \(F_{i-1}\), plus translational velocity added to the frame \(F_i\) due to the motion of the current frame itself. The latter motion can be due to actuation. Using the conventional velocity propagation, and after some considerable derivations we have shown [12] that the translational and rotational velocities of two consecutive frames are related by

\[
\begin{bmatrix}
\dot{u}_i \\
\dot{\phi}_i
\end{bmatrix} =
\begin{bmatrix}
R_{i,j-1} & R_{i,j-1}S_{i,j-1} \\
0 & R_{i,j-1}
\end{bmatrix}
\begin{bmatrix}
\dot{u}_{i-1} \\
\dot{\phi}_{i-1}
\end{bmatrix} +
\begin{bmatrix}
b \\
b
\end{bmatrix}
\begin{bmatrix}
d_i \\
\dot{\lambda}_i
\end{bmatrix}
\]

(1)

where \(S_{i,j-j} = \begin{bmatrix}
0 & -d_i & d_i s_i \\
d_i c_i & 0 & -d_i s_i \\
-d_i s_i & d_i c_i & 0
\end{bmatrix}\) is a skew symmetric matrix, a constant vector, and a rotation matrix, respectively, \(i = 0, \ldots, n\), and \(c\) and \(s\) denote cosine and sine functions respectively. It is noted that \(i = 0\) denotes the rover reference frame, \(i = n - 1\) is the wheeled axil frame and the frames between these two frames are for various linkage and joints connecting the rover body to the wheel axle. The last two frames, i.e. \(n - 1\) and \(n\), will be explained later.

Equation (1) is a fundamental result that relates the motions of the two consecutive frames in terms of certain rover parameters. These parameters are obtained from an extended DH table, referred to here as DHT table. In the DH table, the first four columns are the same as those in DH table, i.e. \((e_i, a_i, \lambda_i, d_i)\), the next two columns consist of \(\dot{\lambda}_i\) and \(\dot{d}_i\), and the joints are identified as either passive or active, and sensed or unsensed. A non-zero \(\dot{\lambda}_i\) or \(\dot{d}_i\) indicates that the corresponding revolute or prismatic joint is variable. A variable joint can be either compliant (passive) or actuated (active). In an articulated rover a variable passive joint is used for compliance of the rover to the terrain. On the other hand an actuated (active) joint can be adjusted as desired. In our DHT table, we show the table entry for \(\dot{\lambda}_i\) or \(\dot{d}_i\) in bold face if it is compliant. Furthermore, any one of the four parameters \((e_i, a_i, \lambda_i, d_i)\) can be sensed/ known or unsensed/unknown. Generally link lengths are known and joint angles are sensed. We show the unknown/unknown quantities in bold face in the DHT. As a result in addition to the four DH parameters, the DHT table identifies joints that are variable or fixed, joints that are passive or active and joints that are sensed or unsensed. These identifications are needed for the derivation of rover kinematics modeling and for actuation to be discussed later.

We can use (1) to cascade the frames from the rover reference to each wheel as [12]

\[
\begin{bmatrix}
\dot{u}_R \\
\dot{\phi}_R
\end{bmatrix} = G_j \begin{bmatrix}
\dot{u}_j \\
\dot{\phi}_j
\end{bmatrix} + H_j \dot{\eta}_j \quad j = 1, 2, \ldots, m
\]

(2)

where \(m\) is the number of wheels, the subscript \(R\) denotes the rover reference, the matrices \(G_j\) and \(H_j\) are, respectively \(6 \times 6\) and \(6 \times 2n_j\) matrices. Equation (2) describes the contributions of individual wheel motions to the rover body motion. The net body motion is the composite effect of motions of all wheels and can be obtained by writing (2) into a single matrix equation as

\[
E \begin{bmatrix}
\dot{u}_R \\
\dot{\phi}_R
\end{bmatrix} = G \begin{bmatrix}
\dot{u}_w \\
\dot{\phi}_w
\end{bmatrix} + H \dot{\eta}
\]

(3)

where \(E = [I_6 \ldots I_6]^T\) is a \(6m \times 6m\) matrix, \(I_6\) is the \(6 \times 6\) identity matrix. The matrices \(G\) and \(H\) are the aggregates of the corresponding matrices in (2). The vectors in (3) are defined as \(\dot{u}_w = \left[\begin{array}{c}
\dot{u}_{c_1} \\
\vdots \\
\dot{u}_{cm}
\end{array}\right]^T\), \(\dot{\phi}_w = \left[\begin{array}{c}
\dot{\phi}_{c_1} \\
\vdots \\
\dot{\phi}_{cm}
\end{array}\right]^T\) and \(\dot{\eta} = \left[\begin{array}{c}
\dot{\eta}_1 \\
\vdots \\
\dot{\eta}_m
\end{array}\right]^T\) where \(\dot{u}_{c_i}\) and \(\dot{\phi}_{c_i}\) are position and orientation at wheel-terrain contact point. Note that some elements of \(\dot{\eta}\) are zero if their corresponding joint values/link lengths are fixed. There is little practical use for the navigation kinematics (3) on its own due to wheel slippage which results in odometry errors. However, (3) is very useful in deriving the actuation kinematics, to be discussed in the following paragraph.

3. Control and Balancing

Motion control of a rover requires determining commands to the actuators so that the rover body moves along a specified trajectory while achieving balanced rover configurations. The latter is required to avoid tipover when the rover traverses on a rough terrain. Actuators consist of wheel and steering motors as well as other motors (linear or rotational) that exist in an articulated high mobility rover.

We can partition the quantities in (3) into two sets; known and unknown. The known are measured (sensed) and specified quantities. Rovers are generally equipped with sensors such as accelerometers for measuring body pitch and roll angles and joint values. The specified quantities are
forward velocity of the rover body $\dot{x}_R$ and its yaw rate $\dot{y}_R$. The specified quantities can also include some wheel slips such as tilt and sway slip rates; $\tilde{\alpha}_j$ and $\tilde{\beta}_j$, $j = 1, \cdots, m$; which are set to zero because of the mechanical construction that does not allow such motions. The unknown quantities consist of actuation quantities that need to be determined for the desired body motion, as well as quantities that are unmeasurable. The actuation quantities are the wheel roll rates $\dot{\theta}_j$, as well as articulated body actuators such as steering that constitute some components of $\tilde{\eta}$ in (3). We identify the known (sensed or specified) by a bar superscript and unknown (i.e. to be found) by a tilde superscript as follows, as

$$
\begin{bmatrix}
\tilde{E} & -\tilde{G} & -\tilde{H}
\end{bmatrix}
\begin{bmatrix}
\tilde{u}_R \\
\tilde{u}_w \\
\tilde{\eta}
\end{bmatrix}=
\begin{bmatrix}
\pi_R \\
\pi_w \\
\eta
\end{bmatrix}
$$

(4)

where $\tilde{E}$ and $\tilde{E}$ are appropriate submatrices of $E$. The submatrices $G, \tilde{G}, H$ and $\tilde{H}$ are similarly defined. Equation (5) can be written in the more compact form of

$$
\tilde{A} \tilde{X} = B \tilde{X}
$$

(5)

where $A = [\tilde{E} - \tilde{G} \, \tilde{H}]$, $B = [-\tilde{E} \, \tilde{G} \, \tilde{H}]$, and $\tilde{X}$ and $\tilde{X}$ are vectors of unknown and known quantities defined in (4). Equation (5) is in the standard form and can be solved for the unknown vector $\tilde{X}$, when a solution exists. The solution to (5) gives $\tilde{X}$ from which the actuation quantities $\tilde{\eta}$ can be extracted.

In general for high mobility rovers there are more unknown than known quantities and the system of equations (5) is underdetermined. This implies that additional requirements can be included to take advantage of the extra degrees of freedom offered by (5). The freedom is brought about due to extra actuators in an active suspension system. We will use the extra freedom to balance the rover configuration as it moves over rough terrain. In such terrain with many bumps and dips, without balance control the rover can loose balance and tip over. We must now define and quantify more precisely the notion of a balanced configuration and express it in terms of rover orientation angles and adjustable joint angles. Stability measures have been suggested before, e.g. [8]. Here, we use wheel-terrain contact position vectors $w_j$, which represent vectors drawn from the rover reference point to the wheel-terrain contact point. Each consecutive pair of vectors (i.e., $w_j$ and $w_{j+1}$) form a plane denoted by $\pi_j$. Let The unit vector perpendicular to this plane is given by $\tilde{s}_j$, and let the unit gravity vector be $\tilde{g}$. Now consider the dot product between unit vectors $s_j$ and $\tilde{g}$, i.e.

$$
\mu_j = \tilde{g}^T \tilde{s}_j
$$

(6)

When the gravity vector $\tilde{g}$ lies in any of the planes $\pi_j$, the vectors $\tilde{g}$ and $s_j$ become orthogonal, resulting in $\mu_j = 0$ and the rover becomes on the verge of tipping over. On the other hand, when the vectors $\tilde{g}$ and $s_j$ are along the same direction, $\mu_j = 1$ for all planes, and the rover is in the most stable configuration. We define the tipover measure as the aggregate of all $\mu_j$, i.e. $\mu = -\prod_{j=1}^{m} \mu_j$. Higher values of $\mu$ corresponds to higher possibility of tipover.

Now we define a balanced rover configuration as one that has a low tipover measure $\mu$, has body pitch $a_R$ and body roll $\beta_R$ close to zero, and is close to its nominal configuration. The latter is a configuration that the rover attains when moving on a flat surface. We must now formulate an objective function whose optimization results in a balanced configuration. Consider minimization of an objective function of the form

$$
f = a_1 \parallel \tilde{\eta}_a - \tilde{\eta}_{an} \parallel + a_2 \alpha^2 + a_3 \beta + a_4 \beta^2
$$

(7)

The vector $\tilde{\eta}_a$ represents the actuated suspension joints which is a sub-vector of the unknown $\tilde{\eta}$, and $\tilde{\eta}_{an}$ is the nominal of $\tilde{\eta}_a$, which is the value when the rover moves on a flat surface. Note that without the fourth term, minimizing $f$ would result in a rover configuration that is maximally flat, i.e. legs spread out even when the rover moves over a flat surface. The second and fourth terms are roll and pitch angles, respectively. Finally the second term is the tipover measure. The weighting factors $a_1, a_2, a_3$ and $a_4$ place relative emphasis between achieving rover balancing and the desire to operate near the nominal configuration.

The balance and motion control problem may be stated as follows. Given the desired rover forward speed $\dot{x}_d$ and heading $\gamma_d$, determine the commands to the wheel and actuated joints, which include the steering, such that the rover maintains the desired forward speed and heading while minimizing the criterion (7). In other words, it is required to minimize (7) subject to (5). The solution to such constrained optimization is [12].

$$
\tilde{X} = A^\# B \tilde{X} - k(I - A^\# A) \begin{bmatrix} \partial f / \partial \tilde{\eta}_a \\ 0 \end{bmatrix}
$$

(8)

where $A^\#$ is the pseudo-inverse of $A$, $k$ is a scalar, $I$ is an identity matrix, $\partial f / \partial \tilde{\eta}_a$ is the vector of the gradient of the performance function with respect to the actuated suspension joints, and the zero vector is of appropriate dimension to make the dimensions compatible. The gradient is computed numerically. The unknown vector $\tilde{X}$ consists of the actuated values and other unknown which such as rolling slip or sides forming a plane denoted by $\pi_j$. Let The unit vector perpendicular to this plane is given by $\tilde{s}_j$, and let the unit gravity vector be $\tilde{g}$. Now consider the dot product between unit vectors $s_j$ and $\tilde{g}$, i.e.
The actuated vector provides the set-points for various motors. Standard controllers, i.e. PID controllers, can then be employed to achieve these set points. Such controllers can be placed on an outer loop and act on the error between the desired rover motion, i.e. forward velocity $\dot{x}_d$ and yaw rate $\dot{\gamma}_d$, and their actual values to keep the rover on the desired trajectory.

We have developed a Matlab program to determine various kinematics forms of a general articulated rover. We have also developed a simulation environment for computing and showing the motion of high mobility rovers over any desired terrain. The terrain is generated by a mathematical function $z = f(x, y)$ specifying the elevation at each point on the terrain. This function can also be generated using a natural terrain image. The rover receives the actuation signal and terrain topology. It then applies the incremental actuation values such as steering, leg actuation and wheel rolling to the rover. The new wheel contact positions and orientations are computed by the rover module. Since the rover wheels must be in contact with the terrain, this module also determines the passive joint angles to ensure proper wheel-terrain contact. After the incremental move and adjustments for the rover to conform to the terrain, the sensed values are sent to the actuation kinematics and balance control for determining the next set of actuations.

4. Applications to an Articulated Rover with Active Suspension

The articulated rover with active suspension (ARAS) to be considered here is similar to the NASA JPL Sample Return Rover shown in Fig. 1. The schematic diagram of ARAS to be analyzed is shown in Fig. 2.

The rover has four wheels with each independently actuated and rotation angles subscripted with a clockwise direction so that $\theta_1, \theta_4$ are for the left side and $\theta_2, \theta_3$ are for the right side. At either side of the rover, two legs are connected via an adjustable hip joint. In Fig. 2 the hip angles on the left and right sides are denoted as $2\sigma_1$ and $2\sigma_2$, respectively. These joints are actuated and used for balancing the rover. The two hips are connected to the body via a differential which has an angle $\rho$ on the left side and $-\rho$ on the right side. On a flat surface $\rho$ is zero but becomes non-zero when one side moves up or down with respect to the other side. The differential joint $\rho$ is passive (unactuated) and provides for the compliance with the terrain. All wheels are steerable with steering angles. We now discuss the results of applying the actuation kinematics and balancing to the ARAS.

4.1 Inclined Surface

The rover is to move down along y-axis on an inclined surface (Fig. 3) then move to the right along x-axis. Initially, the rover is set to its nominal configuration, $\sigma_1 = \sigma_2 = 45$ deg. It starts at $(x, y) = (40 \text{ cm}, 400 \text{ cm})$. The optimization coefficients in (8) are set at $a_1 = 0.6$, $a_2 = 6.0$, $a_3 = 0.6$, $a_4 = 0$. The running time is set to 350 seconds. The hip joint angles keep start increasing from 45 deg, and settle to about 50.5 degrees after 350 seconds, as shown in Fig 4. It results in the stretching configuration in which the rover’s center of mass is lowered to ensure that the tipover will not occur. The roll angle (not shown) remains zero, while the pitch angle (not shown) is same as the slope angle which is about 18 deg. as expected. Subsequently the rover moves along x-axis. The hip angles are shown in Fig. 5 and demonstrate that the rover right side is lowered and its left side is lifted to level the rover and keep the balance to avoid tip over.

![Fig. 1 JPL Sample return rover](image1)

![Fig. 2 Schematics of the rover in Fig. 1](image2)

![Fig. 3 Rover moving down on an inclined surface](image3)
Finally the roll and pitch are shown in Fig. 6 and indicate that the roll angle is reduced from about 20 deg. to about 4 deg. due to the action of hip joints that leveled the rover.

4.2 Wavey Terrain

The selected trajectory is a straight line along x-axis on the wavy terrain of Fig. 4. With wavy terrain, the left shoulder of the rovers may experience the surface with lots of bumps and ditches. On the other hand, the right shoulder experiences a flat surface. The purpose of selecting this trajectory is to understand the behavior of the rover while it is moving over bumps and consecutive sinusoidal waves.

At the beginning, the rover is set to its nominal configuration. It then starts at the point \((x, y) = (80, 140)\), and moves on a straight line along x-axis. The coefficients in (10) are set as \(a_1 = 0.1, a_2 = 2.0, a_3 = 0.1, a_4=0\). The running time is set to 350 seconds. In this case not only the body must be cleared from the terrain bumps, but also the balancing is required.

Fig. 5 shows the hip joint angles over time. These curves occur when the left shoulder goes over a bump and right shoulder moves on a flat surface. The value of right hip joint is always smaller than that of the left one in order to push up the right shoulder. Fig. 6 shows that the body roll angle varies between 0 and 4 degrees which is an improvement, comparing to the initial 18 roll angle. The pitch angle oscillates between -2.5 to 6.2 degrees. This happens because the left side of rover is traversing over sinusoidal surface.
4. Conclusions

We have applied to an articulated rover a previously developed kinematics modeling of high mobility rovers. The method requires only setting up an extended DH table for the rover links and joints. The main feature of the work is its generality, e.g. dealing with both active (actuated) and passive (compliant) joints and linkages, and its ease of implementation. In particular, the proposed formulation makes the computer implementation very efficient. The method is applied to a four wheel rover that has actuated hip joints and compliant differential. It is demonstrated that through balance control, the rover changes its joints angles so as to keep it balanced on uneven terrain. The balance control maintains the rover in a configuration that prevents it from tipover.

References


