Channel Estimation in OFDM System over Wireless Channels

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Abstract—In this paper, we investigate channel estimation techniques for wireless receivers. Channel estimation is important for wireless applications because a wireless channel has fading characteristics and time varying impulse response. OFDM is a multi-carrier modulation technique wherein we use a set of orthogonal sub-carriers. OFDM divides the entire frequency band into several sub-bands. The sub-carriers that are sent with known power are called as pilots. Estimation is based on pilot arrangement. In this paper, we analyze the channel estimation techniques such as LS, MMSE at pilot frequencies for different modulation techniques. A simplified Kalman filter is given which reduces the noise effects of the LS estimation. Also the estimation error in the existing methods is investigated. Here the performance of the techniques is based on the bit error rate and complexity.

Index Terms—Wireless Channel, OFDM, channel estimation, LS, MMSE, Kalman

I. INTRODUCTION

In a classic communication system, the information to be transmitted is modulated onto a single carrier. To obtain high bit rates, the symbols are transmitted faster and hence they occupy the entire bandwidth. When the channel is frequency selective, consecutive symbols will interfere with each other, and thus make it harder to recognize the transmitted symbol. Thus, in a classic communication system, ISI causes severe degradation of the system performance.

Orthogonal Frequency Division Multiplexing modulation technique is an effective way for combating the frequency selective and time varying channels. It has been used widely in wireless communication due to its high speed data transmission and robustness to multi-path fading. A dynamic estimation of the OFDM system is necessary before the demodulation of the transmitted OFDM signal because the wireless channel is frequency selective. Channel estimation can be performed either by inserting pilot symbols into each OFDM symbol or into all the sub-carriers.

In block type pilot channel estimation, the pilots are inserted in all the sub-carriers and the channel is assumed to be slow fading channel. The estimation of channel for block type can be based on Least Square (LS) or Minimum Mean Squared Error (MMSE) approach. MMSE estimator has shown a gain of 10-15dB over LS estimate. Though the LS estimator performance is low compared to that of MMSE, the computational complexity is less [2]. The major system drawback of MMSE, which is complexity, can be eliminated using low rank approximations [1].

II. OFDM SYSTEM MODEL

The serial binary data to be transmitted is first grouped into symbols based on the modulation schemes used, using signal mapper. The OFDM system treats the source symbols i.e. BPSK or QAM symbols at the transmitter as though they are in the frequency domain. Then the serial data is transformed into parallel data. These symbols are used as the inputs to an IDFT block. The IDFT takes N symbols at a time where N is the number of sub-carriers in the system. IDFT operation is performed using IFFT which reduces the number of operation, which in turn gives the time domain signal \( x(n) \).

\[
    x(n) = \text{IDFT}\{X(k)\}, \quad n = 0,1,2,...,N-1
\]

\[
    = \sum_{k=0}^{N-1} X(k) \exp\left(\frac{j2\pi kn}{N}\right)
\]  

The IDFT correlates the frequency domain input data with its orthogonal basis functions, which are sinusoids at certain frequencies. This correlation is equivalent to mapping the input data onto the sinusoidal basis functions. Following the IDFT is the guard band insertion. The guard interval could be all zero samples transmitted before each OFDM symbol. Since it does not contain any useful information it can be removed at the receiver. The guard interval is not used in practical systems since it does not remove the intersymbol interference.

![Baseband OFDM system model](image)

Intersymbol interference can be avoided using cyclic prefix code. Cyclic prefix is a process wherein last symbol of the OFDM is repeated at the beginning which
will allow multipath to settle down before the main data arrives the receiver. Resulting OFDM symbol is given by
\[ x_f(n) = \begin{cases} x(N + n), & n = -N_g, -N_g + 1, \ldots, 0 \\ x(n), & n = 1, \ldots, N - 1 \end{cases} \]
(2)

where \(N_g\) represents the length of guard interval.

Length of cyclic prefix is equal to the guard interval. The multiplexed signal is then sent on through the channel. The channel considered throughout this paper is multipath fading channel with delays. The noise is assumed to be white Gaussian noise and uncorrelated.

\[ y_f(n) = x_f(n)h(n) + n(n) \]
(3)

The time varying impulse response is given by
\[ h(n) = \sum_{i=0}^{r-1} h_i e^{j\frac{2\pi}{N} n f_{nu}} \delta(\tau - \tau_{au}^i) \quad 0 \leq n \leq N - 1 \]
(4)

where \(r\) is the total number of propagation paths, \(h_i\) is the complex impulse response of the \(i^{th}\) path, \(f_{nu}\) is the \(i^{th}\) path Doppler frequency shift, \(\lambda\) is delay spread index, \(T\) is the sample period and \(\tau_{au}^i\) is the \(i^{th}\) path delay normalized by the sampling time.

At the receiver, \(y(n)\) is the convolution of the transmitted OFDM signal with channel impulse response plus the additive noise as shown in Eqn. (3).

\[ Y(k) = DFT\{y(n)\} = \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-j\frac{2\pi}{N} nk} \quad ; \quad k = 0, 1, \ldots, N - 1 \]
(5)

When the entire system is considered,\[ Y(k) = X(k)H(k) + N(k) \]
(6)

where \(H(k)\) and \(N(k)\) are the DFT's of \(h(n)\) and \(n(n)\) respectively.

III. CHANNEL MODEL

The channel model considered is a multipath Rayleigh slow fading channel with time-varying impulse response in which the channel attenuation and excess delay \(\tau_{au}\) remains constant during one symbol duration as described by Eqn. (4). The observed channel impulse response \(h\sqrt{N}\) after sampling the frequency response is given by
\[ h_k = \alpha_m \exp(-j\frac{\pi}{N} k + (N - 1)r_{au}) \sin(\pi r_{au}) \]
(7)

The system described in the Eqn. (3) can be rewritten as,
\[ y_k = x_k h_k + N_k \]
(8)

The corresponding frequency response of the channel is,
\[ H_n = \sum_{k=0}^{N-1} h_k \exp(-j\frac{2\pi}{NT} \tau_{au}) \]
(9)

The transmitted data is estimated by
\[ X_{est}(\frac{\tau}{H}) = \sum_{k=0}^{N-1} h_k \exp(-j\frac{\pi}{N} k + (N - 1)r_{au}) \sin(\pi r_{au}) \]

IV. CHANNEL ESTIMATION

The channel estimation based on pilot arrangement is investigated. The estimation based on pilot can be done using Least Square or Minimum Mean Square Error technique.

Inserting pilot tones in all the sub-carriers at certain periods or into each OFDM symbol can do the channel estimation in OFDM systems either. The former is block type, for which LS and MMSE estimators can be applied, whereas for the later comb type, first estimation is to be performed using LS/MMSE and then the estimated signal is to be interpolated using interpolation techniques such as linear interpolator, second order interpolator, low pass interpolation, spline cubic interpolation and time domain interpolation.

In block type pilot assisted channel estimation, the estimation problem is usually formulation of estimate \(\{h_k\}\) from the received data \(\{Y\}\) and the transmitted data \(\{X\}\). If the channel is constant during a block, there will be no channel estimation error since all the pilots are sent to all the sub-carriers. The channel estimation can be performed through LS/MMSE.

A. LS Estimator

The Least Square estimator attempts to minimize the square of the error between \(X\) and \(Y\).

\[ E[(Y - X)(Y - X)] \]

(10)

The LS estimate for the cyclic impulse response \(\hat{h}\) minimizes \(\|y - XF\|_H\) \(y - XF\) and generates
\[ \hat{h}_{LS} = F^H X^H X F \]
which reduces to
\[ h_{LS} = (X^{-1} Y) \]
(11)

This estimator is equivalent to what is called zero forcing estimators.

B. MMSE Estimator

Here we assume that the channel response as well as the channel noise is uncorrelated Gaussian vectors. Then the MMSE estimate of channel vector becomes \[ h_{mmse} = R_h R_{yy}^{-1} y \]
where
\[ R_{hh} = E[h_{yy}^H] = R_{hh} F^H X H \]
\[ R_{yy} = E[yy^H] = X FR_{hh} F^H X H + \sigma_n^2 I_N \]

\( R_{hh} \) is the auto-covariance matrix of \( h \) and \( \sigma_n^2 \) is the noise variance \( E\{n_k^2\} \).

Then the frequency domain MMSE estimate of \( h_{mmse} \) is given by
\[ \hat{h}_{mmse} = F h_{mmse} \]

\[ = F (R_{yy} R_{yy}^{-1})_Y \]

\[ = F (R_{hh} F^H X H) (X FR_{hh} F^H X H + \sigma_n^2 I_N)^{-1} Y \]

\[ = R_{hh} \left( (F^H X H X F)^{-1} \sigma_n^2 + R_{hh} \right)^{-1} \left( (F^H X H X F)^{-1} \right)^{13} \]

The mean square error for MMSE can be calculated as
\[ E(h - h_{mmse})^2 \]

\[ \text{MSE} = \text{mean} \left( \frac{h - h_{mmse}}{h} \right)^2 \]

Let p denotes first column of the DFT matrix \( k \) and \( R_{hh} \) denotes upper left corner of \( R_{hh} \).

Then the modified MMSE estimator becomes
\[ \hat{h}_{mmse} = PC_{mmse} y \]

\[ = PC_{mmse} P^H X H \]

\[ (14) \]

where
\[ C_{mmse} = R_{hh} \left[ (P X H X P)^{-1} \sigma_n^2 + R_{hh} \right]^{-1} \left( P^H X H X P \right)^{-1} \]

Although the LS estimator modifications do not have much impact, its performance in terms of mean square error can be improved for arrange of SNR’s. LS estimator does not use the channel statistics. Since energy of h outside the first L taps, excluding the low energy taps of \( g \) will improve its performance, whereas the noise energy is assumed to be constant over the entire range[11 13].

The modified LS estimator becomes,
\[ \hat{h}_{LS} = PC_{LS} y \]

The complexity of the modified LS is high when compared to the full LS but modified MMSE complexity is equally complex to that of LS.

Since MMSE is done based on the assumption that channel correlation & noise variance are known, but in practice they are to be taken as constant or found in an adaptive way.

However under special conditions that the channel is T spaced & the energy will not leak outside \([0 L-1]\). In such case the modified MMSE equals the full MMSE, whereas the performance of modified LS dominates the LS estimate Eqn. (15).

### Estimation error:

In this paper we will also evaluate the channel estimation error in the existing method and a new effective channel estimation approach with low estimation error as well.

\( \tau_{au} \) is estimated by either using LS/MMSE methods. The delay \( \tau_{au} \) is assumed to be equal to \( mT_s \). But in practice this assumption is never satisfied [11].

The transmitted signal can be written as,
\[ x(t) = \sum_{n=0}^{N-1} x_n \delta(t - nT_s) \]

The channel estimation algorithms may have estimation error even in the absence of noise. In this section, we will discuss the methods to reduce this type of estimation error.

Based on the excess delay, the channel can be classified into two classes. The first type system model where the channel excess delay is assumed to be equal to mTs/k. For the second model, excess delay is not a rational number.

If a data sample \( x_n \) is up sampled with a factor k, then \( x_r \) be comes \( \{\hat{x}_n\} \) where
\[ x_r \}
\[ 0 \quad \text{Otherwise} \]

\[ x_n \quad \text{if and only if } k = nk \]
which can be represented as,
\[ x(t) = \sum_{k=0}^{N-1} \delta(t-kT_s/N) \]  
(15)

At the receiving end the obtained \( y(t) \) signal is first discretised then down sampled at the sample rate \( k \).
\( y_n = \hat{y}_k \) only if \( n = k/K \)

Let \( Y(f) = F(y_n) \)

By using filter bank concepts [10]
\[ Y(f) = \frac{1}{k} \sum_{k=0}^{K-1} Y(f+k) \]

Discretising we obtain,
\[ y_n = \frac{1}{k} \sum_{k=0}^{K-1} y_n + kN KN \]
\[ H_n(\hat{h}) = \frac{1}{K} \sum_{n=0}^{K-1} \hat{H}_{n+kN} \]

Similarly,
\[ \hat{H}_n = \frac{X_n}{X_0} H_n(\hat{h}) \]

On substituting \( \tau_{au} = mT_s/k \) into Eqn. (3), the channel attenuation is
\[ \hat{H}_n = \sum_{m=0}^{N+1} h_m \exp(-j2\pi km(N)) = \frac{Y_n'}{X_n} \]

It is clear from the Eqn. (16) & (17) that
\[ \hat{H}_n \neq H_n(\hat{h}) \text{ for } 0 \leq n \leq N-1 \]

Suppose that the system has additive white noise, then we have
\[ \hat{Y}_K = X_K \hat{H}_n + N_K \]

where \( \hat{Y} = [\hat{Y}_0 \hat{Y}_1 \hat{Y}_{KN+1}] \) and \( \hat{H} = [\hat{H}_0 \hat{H}_1 \hat{H}_{KN+1}] \)

\[ n = \begin{bmatrix} N0 \\ N1 \\ NKN+1 \end{bmatrix} \]  
and \( h = [h_0 h_1 h_{M-1}] \)

We need to choose a up sampling rate of say \( k \) & discretise the received signal with a sampling rate of \( kT_s \) to get \( y_k \). Once after removing the cyclic prefix from \( y_k \), perform DFT to obtain \( y_k \). Then using LS optimisation problem \( h \) is to be obtained.

The value of \( h \) obtained is substituted to get the channel attenuation \( \{H_n\} \)

Kalman filter for channel estimation in OFDM:

In the proposed algorithm, the Least Square estimate of the channel frequency response at pilot locations can be further improved by individual Kalman smoothing filters to cancel the ICI effects and to reduce noise.

A scalar Kalman filter is applied for a Least-Square estimated value \( H_n \) at \( n \). The filter has an input for receiving \( H_n \), a filter equation and an output for the corrected estimated value \( \hat{H}_n \) for the \( k \)th variable. The filter equation is
\[ H_n^k = K_n^* S_n[k] \]

where \( H_n^k \) is the Kalman estimate of \( H_n \).

Wherein the correction
\[ S_n[k] = S + K_n(H_n - S) \]

and the prediction of the correction is given as
\[ S = K_n S_n[k] \]

Then the Kalman filter gain and minimum prediction MSE are estimated as
\[ K_n = P/(1+P) \]
\[ P = K_n^2 P_n[k] + K_n \]

The constants \( K_n^* \), \( K_n \) and \( K_n \) may be selected as a function of the modulation mode of the sub-carrier. They can be set by evaluating the performance in various multipath fading channels and noise conditions. The filter gain \( K_n \) may also be a constant selected as a function of the modulation mode of the sub-carrier.

D. Simulation Results: Mean Square Error

A considered multi path time varying channel with \( M=5 \) propagating paths is considered.

Figure 4 shows the Mean square error Vs SNR for MMSE and LS estimators. In case of MMSE, for lower SNRs, the approximation effect is small compared to the channel noise, while it becomes dominant for large SNRs. In LS estimator, the Mean Square Error is reduced for large SNRs but same effect as in modified MMSE estimators at significant SNRs. As seen in Fig.4, a gain of 5dB can be obtained using MMSE over LS. Figure 6 shows that the Kalman estimate performs well among the existing channel estimators.
Symbol Error Rate

The SER curves presented are based on MSE of the channel estimators presented in the previous section. For SER calculations we have used formulas from [12].

Given a noisy estimate of the channel these formulas give the SER of BPSK.

CONCLUSION

The channel estimators in OFDM system are investigated. The estimators, Least Square and Minimum Mean Square Error can be efficiently utilised to estimate the channel statistics. MMSE generally requires second order statistics of channel and noise beforehand. Moreover the complexity is large when compared to LS. For large SNRs the LS method works well. Kalman estimator performs well over all the channel estimators. The performance of these techniques are analysed based on mean square error and bit error rate.

REFERENCES


