Software Reliability Growth Model with Logistic-Exponential Testing-Effort Function and Analysis of Software Release Policy

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Abstract- Software reliability is one of the important factors of software quality. Before software delivered in to market it is thoroughly checked and errors are removed. Every software industry wants to develop software that should be error free. Software reliability growth models are helping the software industries to develop software which is error free and reliable. In this paper an analysis is done based on incorporating the logistic-exponential testing-effort in to NHPP Software reliability growth model and also observed its release policy. Experiments are performed on the real datasets. Parameters are calculated and observed that our model is best fitted for the datasets.

Keywords- Software Reliability, Software Testing, Testing Effort, Non-homogeneous Poisson Process (NHPP), Software Cost.

ACRONYMS
NHPP : Non Homogeneous Poisson Process
SRGM : Software Reliability Growth Model
MVF : Mean Value Function
MLE : Maximum Likelihood Estimation
TEF : Testing Effort Function
LOC : Lines of Code
MSE : Mean Square fitting Error

I. INTRODUCTION

Software becomes crucial in daily life. Computers, commutation devices and electronics equipments every place we find software. The goal of every software industries is develop software which is error and fault free. Every industry is adopting a new testing technique to capture the errors during the testing phase. But even though some of the faults were undetected. These faults create the problems in future. Reliability is defined as the working condition of the software over certain time period of time in a given environmental conditions. Large numbers of papers are presented in this context. Testing effort is defined as effort needed to detect and correct the errors during the testing. Testing-effort can be calculated as person/ month, CPU hours and number of test cases and so on. Generally the software testing consumes a testing-effort during the testing phase [20 21].SRGM proposed by several papers incorporated traditional effort curves like Exponential, Rayleigh, and Weibull. The TEF which gives the effort required in testing and CPU time the software for better error tracking. Many papers are published based on TEF in NHPP models [4, 5, 8, 11, 120, 12, 20, 21]. All of them describe the tracking phenomenon with test expenditure.

This paper we used logistic-exponential testing-effort curve and incorporated in the SRGM. The result shows that the SRGM with logistic-exponential

II. SOFTWARE TESTING EFFORT FUNCTIONS

Several software testing-effort functions are defined in literature. w(t) is defined as the current testing effort and W(t) describes the cumulative testing effort. The following equation shows the relation between the w(t) and W(t)

\[ W(t) = \int_0^t w(s) ds \]  

The following are some of them

A. Exponential Testing effort function

The cumulative testing effort consumed in the time (0,t] is [20]

\[ W(t) = N \times (1 - e^{-\lambda t}) \]  

B. Rayleigh Testing effort curve:
The cumulative testing effort consumed in the time \(0,t\) is
\[
W(t) = W(0) + (1 - \exp \left(-\frac{t}{\lambda(t)}\right)) \times \alpha
\]
(3)

The Rayleigh curve increases to the peak and descends gradually with decelerating rate.

C. Logistic-exponential testing-effort:

It has a great flexibility in accommodating all the forms of the hazard rate function, can be used in a variety of problems for modeling software failure data. The logistic-exponential cumulative TEF over time period \((0,t]\) can be expressed as [27]
\[
W(t) = \alpha \times \left(1 + \left(\frac{e^{\alpha x}}{e^{\alpha x} - 1}\right)^{-\beta x}\right)^{\frac{1}{\beta}}, \ t > 0
\]
(4)

And its current testing effort is
\[
w(t) = \alpha \times \left(1 + \left(\frac{e^{\alpha x}}{e^{\alpha x} - 1}\right)^{-\beta x}\right)^{\frac{1}{\beta}}, \ t > 0
\]
(5)

\(\alpha\) is the total expenditure, \(k\) positive shape parameter and \(\lambda\) is a positive scale parameter.

III. SOFTWARE RELIABILITY GROWTH MODELS

A. Software reliability growth model with logistic-exponential TEF

The following assumptions are made for software reliability growth modeling [1, 8, 11, 20, 21, 22]

(i) The fault removal process follows the Non-Homogeneous Poisson process (NHPP)

(ii) The software system is subjected to failure at random time caused by faults remaining in the system.

(iii) The mean time number of faults detected in the time interval \((t, t+\Delta t)\) by the current test effort is proportional for the mean number of remaining faults in the system.

(iv) The proportionality is constant over the time.

(v) Consumption curve of testing effort is modeled by a logistic-exponential TEF.

(vi) Each time a failure occurs, the fault that caused it is immediately removed and no new faults are introduced.

(vii) We can describe the mathematical expression of a testing-effort based on following

\[
\frac{dm(t)}{dt} = \frac{1}{w(t)} = \alpha \times (a - m(t))
\]
(6)

\[
m(t) = a \times \left(1 - e^{-r t} \times (W(t) - W(0))\right)
\]
(7)

Substituting \(W(t)\) into Eq.(7), we get

\[
m(t) = a \times \left[1 - \exp \left(-r t \times \left(\frac{\lambda(t)}{\alpha} - 1\right)^{\frac{1}{\beta}}\right)\right]
\]
(8)

This is an NHPP model with mean value function with the Logistic-exponential testing-effort expenditure. Now failure intensity is given by

\[
\lambda(t) = \frac{dm(t)}{dt} = \alpha \times r \times w(t) \times e^{-r t} \times W(t)
\]
(9)

\[
\lambda(t) = \frac{a e^{-r t} \times \left(1 + \left(\frac{\lambda(t)}{\alpha} - 1\right)^{\frac{1}{\beta}}\right)^{\frac{1}{\beta}}}{1 + \left(\frac{\lambda(t)}{\alpha} - 1\right)^{\frac{1}{\beta}}}
\]
(10)

The expected number of errors detected eventually is

\[
m\infty = \alpha
\]
(11)

B. Yamada Delayed S-shaped model with logistic-exponential testing-effort function

The delayed ‘S’ shaped model originally proposed by Yamada [24] and it is different from NHPP by considering that software testing is not only for error detection but error isolation. And the cumulative errors detected follow the S-shaped curve. This behavior is indeed initial phase testers are familiar with type of errors and residual faults become more difficult to uncover [1, 6, 15, 16]. From the above described section 3.1, we will get a relationship between \(m(t)\) and \(w(t)\). For extended Yamada S-shaped software reliability model. The extended S-shaped model [24] is modeled by

\[
\frac{dm(t)}{dt} = \frac{1}{w(t)} \times \left[m(t) - m(0)\right]
\]
(12)

\[
\frac{dm(t)}{dt} = \frac{1}{w(t)} \times \left[m(t) - m(0)\right]
\]
(13)

We assume \(r_2 > r_1\) by solving 2 and 3 boundary conditions \(m(t=0)\), we have

\[
m(t) = a \times \left(1 - e^{-r t \times W(t)}\right)
\]

and

\[
m(t) = a \times \left(1 - e^{-r t \times W(t)}\right)
\]

At this stage we assume \(r_2 \approx r_1 \approx r\), then using ‘L’ Hospitals rule the Delayed S-shaped model with TEF is given by

\[
m(t) = a \times \left(1 - e^{-r t \times W(t)}\right) \times \alpha \times \left(\frac{\lambda(t)}{\alpha} - 1\right)
\]
(14)

This is an NHPP model with mean value function with the Logistic-exponential testing-effort expenditure. Now failure intensity is given by

\[
\lambda(t) = \alpha \times r \times w(t) \times e^{-r t} \times W(t)
\]
(15)

The failure intensity function for Delayed S-shaped model with TEF is given by

\[
\lambda(t) = \alpha \times r \times w(t) \times e^{-r t} \times W(t)
\]
(16)

IV. EVALUATION CRITERIA

A. The goodness of fit technique

Here we used MSE [5, 11, 17, 23] which gives real measure of the difference between actual and predicted values. The MSE defined as

\[
MSE = \sum_{i=1}^{k} \frac{\left|m_i(t) - m_i\right|^2}{k}
\]
(17)

A smaller MSE indicate a smaller fitting error and better performance.
B. Coefficient of multiple determinations (R2)

Which measures the percentage of total variation about mean accounted for the fitted model and tells us how well a curve fits the data. It is frequently employed to compare model and access which model provies the best fit to the data. The best model is that which proves higher R². that is closer to 1.

C. The predictive Validity Criterion

The capability of the model to predict failure behavior from present & past failure behavior is called predictive validity. This approach, which was proposed by [26], can be represented by computing RE for a data set

$$RE = \frac{(m(t_i) - y_i)}{y_i}$$  \hspace{1cm} (18)

In order to check the performance of the logistic-exponential software reliability growth model and make a comparison criteria for our evaluations [14].

D. SSE criteria:

SSE can be calculated as :

$$SSE = \sum_{i=1}^{n} \left[ y_i - m(t_i) \right]^2$$  \hspace{1cm} (19)

Where $y_i$ is total number of failures observed at a time $t_i$ according to the actual data and $m(t_i)$ is the estimated cumulative number of failures at a time $t_i$ for $i=1,2,.....,n$.

$$PE_i = Actual\{observed\\}_i - Predicted\{estimated\\}_i$$  \hspace{1cm} (20)

$$Bias = \frac{\sum_{i=1}^{n} PE_i}{n}$$  \hspace{1cm} (21)

$$Variation = \sqrt{\frac{\sum_{i=1}^{n} (PE_i - Bias)^2}{n - 1}}$$  \hspace{1cm} (22)

$$MRE = \frac{|M_{estimated} - M_{actual}|}{M_{actual}}$$  \hspace{1cm} (23)

V. MODEL PERFORMANCE ANALYSIS

A. DS1:

The first set of actual data is from the study by Ohba 1984 [15].the system is PL/1 data base application software consisting of approximately 1,317,000 lines of code. During nineteen weeks of experiments, 47.65 CPU hours were consumed and about 328 software errors are removed. Fitting the model to the actual data means by estimating the model parameter from actual failure data. Here we used the LSE (non-linear least square estimation) and MLE to estimate the parameters. Calculations are given in appendix A.

All parameters of other distribution are estimated through LSE. The unknown parameters of Logistic-exponential TEF are $\alpha=72$(CPU hours), $\lambda=0.04847$, and $k=1.387$. Correspondingly the estimated parameters of Rayleigh TEF $N=49.32$ and $b=0.00684/week$. Fig.1 plots the comparison between observed failure data and the data estimated by Logistic-exponential TEF and Rayleigh TEF. The PE, Bias, Variation, MRE and RMS-PE for Logistic-exponential and Rayleigh are listed in Table I. From the TABLE I we can see that Logistic-exponential has lower PE, Bias, Variation, MRE and RMS-PE than Rayleigh TEF. We can say that our proposed model fits better than the other one. In the TABLE II we have listed estimated values of SRGM with different testing-efforts. We have also given the values of SSE, R² and MSE. We observed that our proposed model has smallest MSE and SSE value when compared with other models. The 95% confidence limits for the all models are given in the Table III.

<table>
<thead>
<tr>
<th>TEF</th>
<th>BIAS</th>
<th>VARIATION</th>
<th>MRE</th>
<th>RMS-PE</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESENT</td>
<td>0.2243</td>
<td>1.297</td>
<td>0.033</td>
<td>1.27</td>
</tr>
<tr>
<td>Rayleigh</td>
<td>0.830337</td>
<td>2.169314</td>
<td>0.052676</td>
<td>2.004112</td>
</tr>
</tbody>
</table>

B. DS2:

The dataset used here presented by wood [2] from a subset of products for four separate software releases at Tandem Computer Company. Wood Reported that the specific products & releases are not identified and the test data has been suitably transformed in order to avoid Confidentiality issue. Here we use release 1 for illustrations. Over the course of 20 weeks, 10000 CPU
### TABLE-II
ESTIMATED PARAMETER VALUES AND MODEL COMPARISION FOR DS1

<table>
<thead>
<tr>
<th>Models</th>
<th>a</th>
<th>r</th>
<th>SSE</th>
<th>$R^2$</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRGM with Logistic-exponential TEF</td>
<td>578.8</td>
<td>0.01903</td>
<td>2183</td>
<td>0.9889</td>
<td>128.36</td>
</tr>
<tr>
<td>Delayed S shaped model with Logistic-exponential TEF</td>
<td>353.7</td>
<td>0.08863</td>
<td>7546</td>
<td>0.9615</td>
<td>443.94</td>
</tr>
<tr>
<td>SRGM with Rayleigh TEF</td>
<td>459.1</td>
<td>0.02734</td>
<td>5100</td>
<td>0.974</td>
<td>299.98</td>
</tr>
<tr>
<td>Delayed S shaped model with Rayleigh TEF</td>
<td>333.2</td>
<td>0.1004</td>
<td>15170</td>
<td>0.9226</td>
<td>892.2</td>
</tr>
<tr>
<td>G-O model</td>
<td>760.5</td>
<td>0.03227</td>
<td>2656</td>
<td>0.9865</td>
<td>156.2</td>
</tr>
<tr>
<td>Yamada Delayed S shaped model</td>
<td>374.1</td>
<td>0.1977</td>
<td>3205</td>
<td>0.9837</td>
<td>188.51</td>
</tr>
</tbody>
</table>

### TABLE III
95% CONFIDENCE LIMIT FOR DIFFERENT SELECTED MODELS(DS1)

<table>
<thead>
<tr>
<th>Models</th>
<th>Lower</th>
<th>Upper</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRGM with Logistic-exponential TEF</td>
<td>441.5</td>
<td>716</td>
<td>0.01268</td>
<td>0.02538</td>
</tr>
<tr>
<td>SRGM with Rayleigh TEF</td>
<td>348.6</td>
<td>569.6</td>
<td>0.01651</td>
<td>0.03817</td>
</tr>
<tr>
<td>Yamada Delayed S shaped Model with Logistic-exponential TEF</td>
<td>314.5</td>
<td>392.8</td>
<td>0.07288</td>
<td>0.1044</td>
</tr>
<tr>
<td>Yamada Delayed S shaped Model with Rayleigh TEF</td>
<td>288.7</td>
<td>377.7</td>
<td>0.07507</td>
<td>0.1258</td>
</tr>
<tr>
<td>G-O model</td>
<td>465.4</td>
<td>1056</td>
<td>0.01646</td>
<td>0.04808</td>
</tr>
<tr>
<td>Yamada Delayed S shaped model</td>
<td>343.7</td>
<td>404.4</td>
<td>0.1748</td>
<td>0.2205</td>
</tr>
</tbody>
</table>
that SRGM with logistic-exponential TEF have less MSE than other models.

VI. OPTIMAL SOFTWARE RELEASE POLICY

A. Software Release-Time Based on Reliability Criteria

Generally software release problem associated with the reliability of a software system. Here in this first we discuss the optimal time based on reliability criterion. If we know software has reached its maximum reliability for a particular time. By that we can decide right time for the software to be delivered out. Goel and Okumoto [1] first dealt with the software release problem considering the software cost-benefit. The conditional reliability function after the last failure occurs at time t is obtained by

\[ R(t + \Delta t) = \exp(-[m(t + \Delta t) - m(t)]) \]  

(24)

Taking the logarithm on both sides of the above equation and rearrange the above equation we obtain

\[ \ln R = -m(\Delta t) \times \exp(-r \times W*(t)) \]  

(25)

thus

\[ W*(t) = \frac{1}{r} \left[ \ln m(\Delta t) - \ln \left( \frac{1}{R} \right) \right] \]  

(26)

By solving the eq 26 we can reach the desired reliability level. For DS1 \( \Delta t=0.1 \) R=0.91 at T=42.1 weeks

B. Optimal release time based on cost-reliability criterion

This section deals with the release policy based on the cost-reliability criterion. Using the total software cost evaluated by cost criterion, the cost of testing-effort expenditures during software testing/development phase and the cost of fixing errors before and after release are: [9, 13, 25]

\[ C(T) = C_1 m(T) + C_2 [m(T_{LC}) - m(T)] + C_3 \int_0^T w(x) \, dx \]  

(27)

Where \( C_1 \) is the cost of correcting an error during testing, \( C_2 \) is the cost of correcting an error during the operation, \( C_2 > C_1 \), \( C_3 \) is the cost of testing per unit testing effort expenditure and \( T_{LC} \) is the software life-cycle length.

From reliability criteria, we can obtain the required testing time needed to reach the reliability objective \( R_0 \). Our aim is to determine the optimal software release time that minimizes the total software cost to achieve the desired software reliability. Therefore, the optimal software release policy for the proposed software reliability can be formulated as Minimize \( C(T) \) subjected to \( R(t + \Delta t) \geq R_0 \) for \( C_1 > C_2, C_3 > 0, \Delta t > 0, 0 < R_0 < 1 \).

Differentiate the equation (20) with respect to \( T \) and setting it to zero, we obtain

\[ \frac{d}{dT} \left( C(T) + C_1 \left( \frac{d}{dT} m(T) \right) + C_2 \left( \frac{d}{dT} m(T_{LC}) - \frac{d}{dT} m(T) \right) + C_3 w(T) \right) \]  

(28)

\[ \frac{d}{dT} m(T_{LC}) = 0 \]  

(29)

\[ \frac{d}{dT} C(T) = C_1 \left( \frac{d}{dT} m(T) \right) + C_2 \left( \frac{d}{dT} m(T) \right) + C_3 w(T) = 0 \]  

\[ \frac{d}{dT} m(T) = \lambda(T) \]  

\[ \frac{\lambda(T)}{w(T)} = \frac{c_3}{c_2 - c_1} \]  

\[ a r e^{-r W(t)} = r \]  

(30)

When \( T=0 \) then \( m(0)=0 \) and \( \frac{\lambda(T)}{w(T)} = a r \) \( \lambda(T) \)

When \( T\to\infty \), then \( m(\infty) = a \)

\[ \lambda(T) \]  

\[ \frac{\lambda(T)}{w(T)} = a \times r \times e^{-r \times t} \]  

(31)

therefore \( \frac{\lambda(T)}{w(T)} \) is monotonically decreasing in \( T \).

To analyze the minimum value of \( C(T) \) Eq. (27) is used to define the two cases of \( \frac{\lambda(T)}{w(T)} \) at \( T=0 \).

1) if \( \frac{\lambda(0)}{w(0)} = a \times r \leq \frac{C_3}{C_2 - C_1} \) , then

\[ \frac{\lambda(T)}{w(T)} \leq \frac{C_3}{C_2 - C_1} \]  

for \( 0 < T < T_{LC} \) it can be obtained at \( dC(T)/dT > 0 \) for \( 0 < T < T_{LC} \) and the minimal value can found at \( C(T) \) can be found at \( T=0 \).

\[ a \times r e^{-r W(t)} = r (a - m(T)) \]

\[ \frac{\lambda(0)}{w(0)} = a \times r \times e^{-r \times t} \times a \]

there can be found a finite and unique real number \( \frac{\ln(\frac{C_3}{C_2 - C_1})}{r} \)

(31)

\[ \frac{\lambda(0)}{w(0)} : \frac{C_3}{C_2 - C_1} \times \frac{\lambda(T)}{w(T)} = \frac{a \times r \times e^{-r \times t} \times a}{a \times r \times e^{-r \times t} \times a} \]

because \( dC(T)/dT < 0 \) for \( 0 < T < T_0 \) and \( dC(T)/dT > 0 \) for \( T > T_0 \) , the minimum of \( C(T) \) is at \( T=T_0 \) for \( T_0 \leq T \) we can easily get the required testing time needed to reach the reliability objective \( R_0 \) here our goal is to minimize the total software cost under desired software reliability and then the optimal software release time is obtained. That is can minimize the \( C(T) \) subjected to \( R(t + \Delta t) \geq R_0 \) where \( 0 < R_0 < 1 \) [9,25]

\[ T^* = \text{optimal software release time or total testing time} = \text{max} \{T_0, T_1\} \]  

(31)

\[ T_1 = \text{finite and unique T satisfying Eq.(31)} \]

\[ T_2 = \text{finite and unique T satisfying Eq.(31)} \]  

By combining the above analysis and combining the cost and reliability requirements we have the following...
Theorem: Assume $C_2 < C_1 < 0$, $C_3 < 0$, $\Delta t > 0$, and $0 < R_0 < 1$. Let $T^*$ be the optimal software release time

A. if \[
\frac{\lambda(0)}{w(0)} > \frac{C_3}{C_2 - C_1}
\]

and \[
\frac{\lambda(T)}{w(T)} = a \times r \times e^{-r \times \alpha} \leq \frac{C_3}{C_2 - C_1}
\]

then

\[
T^* = \begin{cases} 
T_0 & \text{for } 0 < R_0 < \left(\frac{\Delta t}{0}\right) \\
T_1 & \text{for } \left(\frac{\Delta t}{0}\right) < R_0 < 1 \\
0 & \text{for } R_0 \geq 1
\end{cases}
\]

B. if \[
\frac{\lambda(0)}{w(0)} \geq \frac{C_3}{C_2 - C_1}
\]

then \[
T^* = \begin{cases} 
T_1 & \text{for } \left(\frac{\Delta t}{0}\right) < R_0 < 1 \\
0 & \text{for } R_0 \geq 1
\end{cases}
\]

C. if \[
\frac{\lambda(0)}{w(0)} \leq \frac{C_3}{C_2 - C_1}
\]

then \[
T^* = \begin{cases} 
T_1 & \text{for } \left(\frac{\Delta t}{0}\right) < R_0 < 1 \\
0 & \text{for } R_0 \geq 1
\end{cases}
\]

From the first dataset estimated values of SRGM with Logistic-exponential testing effort function $\alpha = 72$ (CPU hours), $\lambda = 0.04847$ /week, $k = 1.387$, $a = 578.8$ and $r = 0.01903$ when $\Delta t = 0.1$, $R_0 = 0.85$ and we let $C_1 = 2$, $C_2 = 50$, $C_3 = 150$ and $T_{LC} = 100$ the estimated time $T_1 = 37.1$ weeks and release time from Eq 30 $T_0 = 39.5$ weeks. Now optimal Release Time max ($37.1, 39.5$) is $T^* = 39.5$ weeks. Fig 10 shows the change in software cost during the time span. Now total cost of the software at optimal time 8354.

From the second dataset estimated values of SRGM with Logistic-exponential testing effort function $\alpha = 12600$ (CPU hours), $\lambda = 0.06352$ /week, $k = 1.391$, $a = 135.6$ and $r = 0.0001432$ when $\Delta t = 0.1$, $R_0 = 0.85$ and we let $C_1 = 2$, $C_2 = 200$, $C_3 = 2$ and $T_{LC} = 100$ the estimated time $T_1 = 18.1$ weeks and release time from Eq 31 $T_0 = 39.5$ weeks. Now optimal Release Time max ($8.05, 18.1$) is $T^* = 18.1$ weeks. Fig 11 shows the change in software cost during the time span. Now total cost of the software at optimal time 20,100.

CONCLUSION

In this paper, we proposed a SRGM incorporating the Logistic-exponential testing effort function that is completely different from the logistic type Curve. We observed that most of software failure is time dependent. By incorporating testing-effort into SRGM we can make realistic assumptions about the software failure. The experimental results indicate that our proposed model fits fairly well.

REFERENCES


