Design and Analysis of H-infinity Controller for a Hypersonic Wind Tunnel

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Abstract. This paper presents a control algorithm for the regulation of pressure inside the settling chamber of a hypersonic wind tunnel, which is a test facility used to generate hypersonic flow regime. Regulation of pressure inside the settling chamber is the most important task for its efficient operation. Here the open loop response of the linear model of a hypersonic wind tunnel incorporating the effect of air pressure is determined. An h-infinity controller is designed by properly selecting the weighing function that improves the settling time.

Keywords: Hypersonic wind tunnel, Pressure vessel, Pressure regulation, settling chamber pressure, H-infinity controller.

1 Introduction

Wind tunnels are used to study the effects of air moving past the specimen under test. The pressure variation studies are based on the flight paths of the space vehicles. The speed is indicated by Mach number which is defined as the ratio of speed of aircraft to speed of sound in gas. The main parts of a hypersonic wind tunnel (mach number >5) are high pressure system, pressure regulating valve, heater, settling chamber, nozzle and test section as shown in fig. (1) [1], [2]. Compressed air from the air storage tank is released through a pressure valve to the heater where it is heated to the required temperature and is straightened in the settling chamber and passed to the test section through the nozzle.

![Fig. 1. Block diagram of a hypersonic wind tunnel system](image)

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The system performance is decided by the speed of settling chamber pressure and is accurately controlled by modelling and designing a suitable \( h\)-infinity controller. For modelling, the continuity equations and parameter values are selected for the vessels \([2]\) - \([6]\). The state space model of the system is given in equation \((1)\) \& \((2)\).

\[
\begin{bmatrix}
\dot{P}_1 \\
\dot{P}_2 \\
\dot{P}_3 
\end{bmatrix}
= \begin{bmatrix}
-K_1/C_1 & 0 & 0 \\
-K_3/C_2 & -K_4/C_2 & -K_n/C_3 \\
0 & K_3/C_3 & K_4-K_n/C_3 
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2 \\
P_3 
\end{bmatrix}
+ \begin{bmatrix}
-K_2/C_1 \\
K_2/C_2 \\
0 
\end{bmatrix} m. 
\]

\( (1) \)

\[
Y_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2 \\
P_3 
\end{bmatrix}
\]

\( (2) \)

Where \( P_1, P_2 \) are the upstream and downstream pressures, \( P_3 \) is the settling chamber pressure, \( m \) is the stem movement of pressure valve, \( K_1, K_2, K_3 \) and \( K_4 \) are constants, \( K_n \) is the nozzle flow constant and, \( C_1, C_2, C_3 \) represents the capacitance of the three pressure vessels respectively.

The system is linearized and the transfer function \([2]\) is given in equation \((3)\).

\[
G_p(s) = \frac{-2.369e006s^2 + 7.897e007s + 4.21e005}{0.015s^5 + 0.7802s^4 + 9.89s^3 + 18.46s^2 + 3.377s + 0.01937}.
\]

\( (3) \)

## 2 Open Loop Response of the Linear Model

The system is simulated using Matlab and the open loop response of settling chamber pressure is plotted in fig. (2). It is observed that the chamber pressure takes 450sec to settle down which is quite high when the test duration is very short. In order to achieve robust performance and fast stabilization, an \( h\)-infinity controller is designed and the performance is evaluated.

![Fig. 2. Open loop response of pressure \( P_3 \) of the linear model](image)
3 Design and Analysis of H-infinity Controller

H-infinity is defined as the space of proper and stable transfer functions. The objective is to minimize the H-infinity norm which is the energy gain of the system. Standard feedback configuration with weights [5], [6] is given in fig. (3). The controller is designed by properly selecting the weighing functions [5]. Here G is the plant transfer function, \( G_d \) the transfer function corresponding to input disturbance, \( r \) the set point, \( u \) the actuator, \( v \) the sensor measurement, \( K \) the controller, \( d \) the disturbance, \( n \) is measurement noise, \( Z_1 \) is the settling chamber pressure, \( Z_2 \) is control output, weight \( W_p \) is the second order transfer function and is selected such that \( |S(j\omega)| < \frac{1}{W_p(j\omega)} \), \( \forall \omega \) where \( S \) is the sensitivity function. Weight \( W_u \) indicates control input weight and sensor noise effects are \( W_n \) [7].

![fig3](image)

Fig. 3. Standard feedback configuration with weights

3.1 Design of weighing functions \( W_u, W_p, W_n \)

The multiplicative uncertainty weight \( W_u \) are selected by satisfying the stability conditions [7] - [9].

\[
|W_u(j\omega)| \geq l_u(\omega), \quad \forall \omega.
\]  

(4)

where \( l_u \) is the relative error of the plant transfer function. The bode plot of plant transfer function and weighing function \( W_u \) is shown in fig.(4). \( W_u \) is selected such

![fig4](image)

Fig.4. Stability criterion for the selection of weight, \( W_u \).
that the magnitude of weight transfer function lies above the plant transfer function.

$$W_w = \frac{600s + 210}{20s + 0.0001}.$$  \hspace{0.5cm} (5)

The sensitivity function $S(s)$ [7] - [9] is defined as

$$S(s) = (1 + K(s)H(s))^{-1}. \hspace{0.5cm} (6)$$

The performance requirement is guaranteed if and only if the condition

$$|S(j\omega)| < \frac{1}{W_p(j\omega)}, \hspace{0.5cm} \forall \omega$$

is satisfied.

The nominal performance criterion [10] is given in equation (7).

$$|W_p(j\omega)| < |1 + G_m(j\omega)|, \hspace{0.5cm} \forall \omega. \hspace{0.5cm} (7)$$

The bode plot of $S(j\omega)$ and $W_p(j\omega)$ and $|1 + G_m(j\omega)$ and $W_p(j\omega)$ are shown in fig.(5)(a) and (5)(b) respectively. It is observed that $1/W_p(j\omega)$ is greater than $|S(j\omega)|$ in fig.(5)(a) and $|W_p(j\omega)|$ is less than $|1 + G_m(j\omega)|$ in fig.(5)(b).

Fig.5. Nominal performance criterion for the selection of weight, $W_p$.

The robust performance is defined by criterion $|W_p(j\omega)S_p(j\omega)| < 1, \hspace{0.5cm} \forall S_p, \omega$ and its bode plot is shown in fig.(6). It is clear from figure that the product $|W_p(j\omega)S_p(j\omega)|$ is less than unity.

Fig.6. Robust performance criterion for the selection of weight, $W_p$. 

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The weighing function $W_p$ is selected satisfying the above conditions as

$$W_p = \frac{30s + 20}{20s + 1}. \quad (8)$$

The weighing function $W_n$ is chosen by trial and error method as

$$W_n = \frac{1}{10}. \quad (9)$$

After designing the three weights $W_u, W_p, W_n$, the $\infty$-controller is simulated in Matlab, with the input disturbance transfer function $G_d = 1$ and the set point equal to $70*10^5$ Pa.

4 Results and Discussions

From the simulation, the $\infty$-controller matrix, $K$ is obtained as

$$K = (1.0e + 005)^* \begin{bmatrix}
-2.0733 & 1.6054 & 1.1876 & 0.2099 & -0.168 & -0.0007 & 0 & 0 & 0.0001 \\
-1.8229 & 1.4115 & 1.0442 & 0.1845 & -0.1291 & -0.0006 & 0 & 0 & 0 \\
-1.3565 & 1.0503 & 0.7770 & 0.1373 & -0.0960 & -0.0004 & 0 & 0 & 0 \\
0.9738 & -0.7540 & -0.5578 & -0.0987 & 0.0689 & 0.0003 & -0 & 0 & 0 \\
-0.2895 & 0.2241 & 0.1658 & 0.0294 & -0.0209 & 0.0003 & 0 & 0 & 0 \\
0.0008 & -0.0007 & -0.0005 & -0.0004 & -0.0001 & -0.0000 & 0 & 0 & -0.0001 \\
-0.0035 & 0.0027 & 0.0020 & 0.0004 & -0.0003 & -0.0000 & 0 & 0 & 0 \\
-0.0001 & 0.0001 & 0.0000 & 0.0000 & -0.0000 & -0.0000 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\infty
\end{bmatrix}$$

The settling chamber pressure with $\infty$-controller when the input disturbance transfer function, $G_d$ is made equal to 1 is shown in fig. (7). The chamber pressure is reduced to 70 bar in 2 sec and is 71 bar in 17 sec where as it is 450sec in the case of open loop response. The results show that $\infty$-controller gives drastic reduction in the settling time thereby improving the transient response to a great extent. This promises greater speed in testing of settling chamber pressure for the specimen placed in hypersonic flow regime. It is also of great advantage that the order of $\infty$-controller is same as that of augmented plant that results in increased computational speed.

![Fig. 7. Settling chamber pressure with set point 70*10^5 Pa for G_d = 1.](image-url)
5 Conclusions

An h-infinity controller is designed for the linear model of the high pressure system of the wind tunnel for regulating the settling chamber pressure. It is designed by properly selecting the weight functions and tuning sensitivity and complementary sensitivity functions. H-infinity controller is simulated in Matlab using the designed weights with input disturbance transfer function $G_d$ made equal to 1 and the set point equal to $70*10^5$ Pa. The controller output performance is compared with the open loop response of the system. It is observed that the pressure in the chamber settles within 2 seconds where as it takes 450 seconds in open loop. This shows that the system performance is improved to the required speed thereby satisfying the specifications. The system accuracy can further be improved by incorporating nonlinearities into the model and selecting appropriate control strategy. Research in this direction is progressing.

References