Investigating Correlation Properties of Hodgkin-Huxley Model with Leaky Integrate–and-Fire Model

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Abstract—Behavioral study of linear model i.e. Integrate and fire model is compared with non-linear model i.e. Hodgkin-Huxley model. Both the models output are affected by Poisson inputs that are correlated positively. Responses of Integrate and fire model and Hodgkin-Huxley models differ in terms of correlated inputs. Linear Integrate and fire model is better for correlated inputs. Non-Linear model displays beneficial qualities of both Hodgkin-Huxley and linear Integrate and fire models. In Integrate and fire model, neurons fire faster and more irregularly when the soma is smaller. In Hodgkin-Huxley model, neurons fire faster with increase in the strength of inhibitory inputs. The effect of inhibitory inputs is considered on both the models.

Index Terms—Leaky integrate and fire model, Hodgkin-Huxley model, correlated inputs, Post synaptic potentials, Behavioural study.

I. INTRODUCTION

We are presenting the differences between Hodgkin Huxley and Integrate and Fire model only at abstract level. Modelling of neurons takes place in Integrate and Fire model and biophysical mechanism of cells is described in Hodgkin-Huxley (HH) model. From Ref. [1], even when inputs are exclusively excitatory, different spike trains become irregular when $p = 0.1$. Smaller the value of $p$ higher is the $C_v$ of different spike trains. $C_v$ is independent of blockage of inhibitory inputs. Ref. [1, 4] most results show that inputs are independent both spatially and temporally in case of IF and HH models. Ref. [5, 6] clearly shows that nearby neurons usually fire in a correlated way. So, former assumption contradicts the latter one. Ref. [7] it also states the principle which states that similar function neurons group and fire together. Under high coefficient of variation ($C_v$), both Complex spikes and Simple spikes trains have been described as highly irregular. In Ref. [8] Analysis of neuronal model behaviour of correlated inputs takes place. From Ref. [3] before comparisons can be made between spiking models, adjustments to model parameters are necessary. Linear IF model behaviour will be different from Non-Linear HH model. Unlike IF model, correlation in input signals has opposite effect on $C_v$ of HH model Ref. [10]. $C_v$ of different spike trains reduces with the correlated inputs. Hence, both the models operate in different modes. Ref. [15] excellent description of electrical behaviour of neurons is provided in the framework of HH conductance based models. HH model increment is obtained on signal to noise ratio (SNR) with increase in input correlation whereas decrement in Signal to Noise ratio (SNR) is obtained for IF model. Response surface methodology is to show graphically different behaviour between two models. It comprises a group of statistical techniques for empirical model building and model exploitation. This method is proposed to show the correlations between output activities

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of two models.
This paper is organized as follows. In Sec. 2 Formulation of above mentioned models are introduced. Sec. 3 is devoted to Numerical Analysis of both the models. In this section behaviour of models with correlated inputs is shown. Sec. 4 is all about Discussions.

II. FORMULATION OF MODEL EQUATIONS.

We consider HH model where evolution of the voltage across the neuron’s membrane is governed by a differential equation:
\[ C \frac{dV}{dt} = -gNa m^3 h (V - V_{NA}) dt - gK n^4 (V - V_K) dt - gL (V - V_L) dt + dI_{syn}(V, T) \]
The parameters used an above equation are: 
\[ C = 1 \mu F/cm^2 \] is the capacitance of the membrane, 
\[ gNa = 120 \text{ mS/cm}^2 \] and 
\[ gK = 36 \text{ mS/cm}^2 \] are the maximal conductance associated with sodium and potassium channels. The leakage conductance is 
\[ gL = 0.3 \text{ mS/cm}^2 \] which is constant. Reversal potentials are 
\[ V_K = -77 \text{ mV}, V_{NA} = 50 \text{ mV} \] and 
\[ V_L = 54.4 \text{ mV} \]. 
\[ m, h \] and \[ n \] are the activation variables.

The time evolution of activation variables depends on the voltage dependent rate constants. During action potential, rate constants shows sharp transitions as a consequence of rapid changes in membrane voltage. Where the experimentally determined voltage transition rates are given explicitly by the expressions
\[ \frac{dn}{dt} = \frac{n_{\infty} - n}{\tau_n} \]
\[ \frac{dm}{dt} = \frac{m_{\infty} - m}{\tau_m} \]
\[ \frac{dh}{dt} = \frac{h_{\infty} - h}{\tau_h} \]
And \( n_{\infty}, m_{\infty} \) and \( h_{\infty} \) are steady state values for activation and inactivation.

The outgoing spikes interspike interval (ISI) is:
\[ T = \inf \{ t : V_t \geq V_{thre} \} \]
III. NUMERICAL ANALYSIS.

For IF model with or without reverse potential, we consider how correlated inputs affect the variability of cellular output. For both the variability of spike trains measured by $Cv$ is non-decreasing function of input correlation. Ref. [2] $Cv$ of the IF model is above 0.5 when the correlation coefficient is greater than 0.09, no matter how strong inhibitory inputs are $Cv$ for IF model with reverse potential is always greater than 0.5 when correlation coefficient is greater than 0.5, independent of strength of inhibitory inputs.

We have shown that multiple spikes are generated with current value between 0.1 and 1 in mat lab. In Ref. [11] Leaky Integrate and Fire Model, for equation

$$V = V - \left( \frac{V}{R \times C} \right) + \left( \frac{I}{C} \right)$$

Value of the graph varies with the change in the value of current. With increase in current, number of spikes increases. For value of current 0.1 we have a spike in graph as shown in figure 1(a). The similar kind of spike is obtained for values of current between 0.1 to 0.3. From 0.4 values we get a spike 1(b) and for value $\geq$ 0.6 we get multiple spikes 1(c). But in case of HH model, right from first value of current we get multiple spikes. These spikes increase with increase in the value of current. For first value i.e. $I = 0.1$ we get two spikes as in figure 2(a) and for $I = 0.3$ we get three spikes as shown in figure 2(b).

For HH model the stronger the input correlation the more regular is the output. The IF model works in an environment of less correlated inputs as it is sensitive to its input correlation. Ref. [1] the most important difference between the two models is that HH model has a refractory period of about 12.2 msec.

Figure 1 (a) When an input current is applied, the membrane voltage increases with time until it reaches a constant threshold $V_{th}$, at which point a delta function spike occurs and the voltage is reset to its resting potential.

Figure 1 (b) After voltage reset the model continues to run. It forms a spike in a linear fashion. In this graph after reaching a certain threshold value $V_{th}$ there is decay in spike.
Figure 1 (c) The firing frequency of the model thus increases linearly without bound as input current increases. We thus observe linear increase in membrane potential with increase in value of the current.

Figure 2 (a) Changing the value of Applied current will change value of function \( y \) resulting in formation of non-linear graph. X-axis represents time and y-axis is function \( Y \), where

\[
\frac{dy}{dt} = \left( \frac{1}{C \text{m}} \right) \left( I - (I \cdot Na + I \cdot k + I \cdot l) \right); \quad a_n(V) \cdot (1 - n) - b_n(V) \cdot n; \quad a_m(V) \cdot (1 - m) - b_m(V) \cdot m; \quad a_h(V) \cdot (1 - h) - b_h(V) \cdot h
\]

Figure 2 (b) Similarly as current increases no. of spikes increases non-linearly. Taking parameters \( V = -60 \text{ Mv} \), \( n = 3.17 \), \( m = 0.0529 \), \( h = 0.001 \), \( C \text{m} = 0.01 \).

We have presented works of both the models. For generation of action potential, Hodgkin-Huxley model is constructed by writing the membrane current as a sum of the leakage current, a delayed rectifier \( k^- \) current and a transient \( Na^- \) current which are the total current flowing across the membrane. Here, \( n, m \) and \( h \) are gating variables. Ref. [16] these variables are used for opening and closing of gates in modeling channels. Using equation:

\[
im = \overline{g}_L(V - E_L) + \overline{g}_K n^4 (V - E_K) + \overline{g}_N A m^3 h (V - E_N A)
\]
The channel consists of both activation and inactivation gates. When positive ions leave the neurons, the membrane current is said to be positive and vice versa. In Fig. 3, we take Hodgkin-Huxley model which is a non linear model because of non-linear differential equation which is shown above. To prove this fact graphically, we plot a graph between $V$ i.e. voltage and $im$ i.e. membrane current. We take different values for $V$ and rest of the parameter values will be constant as mentioned below the figure. This way multiple values for $im$ is obtained and graph is plotted which shows that as voltage decreases, membrane current changes non-linearly.

For IF model, when membrane potential of a neuron reaches a threshold value of about -55 to -50mv then a neuron typically fires an action potential. During action potential, path followed by membrane potential is rapid, stereotyped trajectory. After action potential, potential is reset to a value $V_{reset}$ below threshold potential $V_{reset} < V_{th}$.

The interspike interval can be determined for constant $I_e$, which we call the interspike-interval firing rate of the neuron:

$$risi = \frac{1}{t_{isi}} = \frac{ln(Rmle + EL - V_{reset})}{Rmle + EL - V_{th}}$$

This expression is valid only when $Rmle > V_{th} - EL$; else $risi = 0$.

In Fig. 4 we have considered Integrate–and-Fire Model which is Linear Model. We have shown this fact graphically below. Plot a graph between $le$ and $risi$. Taking different values for $le$ and rest of the parameters will remain same as mentioned below the figure. We get varying values for $risi$ i.e. interspike interval rate and plot the graph which shows that values of $risi$ increases linearly with increase in the value of $le$.

![Figure 3](image1.png)

**Figure 3**: The maximal conductance and reversal potentials used in the graph are $g_L=0.003$ ms/mm$^2$, $g_K=0.36$ ms/mm$^2$, $g_Na=1.2$ mm/ms$^2$, $EL=-54.387$ mV, $EK=-77$ mV, $ENA=50$ mV.

![Figure 4](image2.png)

**Figure 4**: The line gives $risi$ for a model neuron with $EL=-65$mV, $V_{reset}=-50$mV, $Rm=90$ MΩ, $tm=30$ ms.
IV. DISCUSSIONS AND CONCLUSIONS.

In Ref. [1], it is reported that the IF model and HH model behave differently as independent inputs to neuronal models are considered. In this paper we present that both models behave totally in different manner in case of correlated inputs. Major difference shown is that in Leaky integrate and fire model we start getting multiple spikes from value of current 0.4 and in HH model we get multiple spikes right from current value 0.1. The SNR of latter increases with increase in input correlation and former decreases. The key difference is that in IF model leakage coefficient is constant but in case of HH model it depends on membrane potential. From Ref. [13] when dealing with task of different complexity the brain might use different coding strategies. Ref. [14] HH model gives an excellent approximation of interspike intervals at different firing frequencies, as well as rise and decay of membrane potential. IF model gives poor approximation when taken under same condition.

IF model is mathematical description of neurons whereas HH model is itself a mathematical model. Former is designed to describe and predict biological processes but the latter one describes initiation and propagation of action potential in neurons. The Hodgkin-Huxley framework of conductance-based models provides full details of the electrical properties of neurons. Whereas the broad features that biological neurons share are captured by integrate and fire model. As more details have been learned about physiology of neurons the HH model is a “modeling schema” rather than just a “model”. Various graphs have been plotted to show differences lying in linearity of both the models.

On the basis of the work presented in Sec. 3, we conclude that For IF model which is linear, we naturally expect that an increase in Input mean or variance results in increase in the output mean or variance. For HH model, which is non-linear it is not always true. In fact it is well proved in some literature that firing rate of HH model increases with increase in inhibitory inputs, in certain cases. In Fig. 1 and Fig. 2, comparison is done between the generation of multiple spikes in Hodgkin-Huxley and Leaky Integrate-and-Fire Models.

We conclude that multiple spikes in case of LIF Model are obtained from value≥0.6 whereas in HH Model it is obtained from value≥0.1. From Fig. 3 and Fig. 4 we conclude graphically that Hodgkin-Huxley Model is Linear and vice-versa.

ACKNOWLEDGEMENT

I wholeheartedly thank Mrs. Jyotsna Singh for the guidance.

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