BER Comparison of $4 \times 4$ and $8 \times 8$ Alamouti MIMO Systems in the Presence of Channel Estimation Errors

Shailendra Kumar Mishra$^1$ and Vidhyacharan Bhaskar$^2$

Dept. of ECE, SRM University, Kattankulathur, Kancheepuram, Tamil Nadu, India.  
e-mail: $^1$sant10287@gmail.com; $^2$vcharan@gmail.com

Abstract. Multiple-Input Multiple-Output (MIMO) systems have attracted great interest since they can improve channel capacity and reliability of wireless communications. However, adopting a MIMO system increases system complexity and cost of implementation. This paper will be dealing with receiver antenna selection to reduce implementation complexity. Space Time Sum of Squares (STSoS) combining selection diversity is used, which has much simpler implementation and provides improved Bit Error Rate (BER) performance. The effects of channel estimation errors on this selection scheme are examined. The BER of Binary Phase-Shift Keying (BPSK) in Rayleigh fading using Alamouti transmission scheme and receiver selection diversity in the presence of channel estimation error is discussed. Numerical analysis is done for different number of transmit and receive antennas and graphs are plotted showing comparison among all.

Keywords: Alamouti transmission scheme, estimation error, multiple input multiple output (MIMO), selection combining scheme, space time block code.

1. Introduction

Various schemes that employ multiple antennas at the transmitter and receiver are being considered to improve the range and performance of communication systems. By far the most promising multiple antenna technology happens to be the so called Multiple Input Multiple Output (MIMO) system. MIMO systems employ multiple antennas at both the transmitter and the receiver.

The single most effective technique to accomplish reliable communication over wireless channels is diversity, which attempts to provide the receiver with independently faded copies of the transmitted signal with the hope that at least one of these replicas will be received correctly. Various diversity techniques can be combined to further improve system performance in a wireless environment [1].

There is an unprecedented growth in the demand for providing reliable high speed wireless communication links in order to support a wide range of applications, including voice, video, e-mail, web browsing, to name a few. Providing such reliable links is challenging due to the fact that in a
wireless environment, unlike many other channels, transmitted signals are received through multi-paths which usually add destructively resulting in serious performance degradation. This phenomenon is normally referred to as fading. Other challenges for high speed wireless applications include scarcity of available bandwidth, highly constrained transmit powers, as well as hardware complexity and cost requirements.

The single most effective technique to accomplish reliable communications over a wireless channel is diversity, which attempts to provide the receiver with independently faded copies of the transmitted signal with the hope that at least one of these replicas will be received correctly. Diversity combining is the technique applied to combine multiple received signals of a diversity reception device to obtain a single improved signal. Space time Block Coding (STBC) is a simple yet very effective means to achieve transmit diversity when other forms of diversity may be limited or non-existent. Such codes can be easily generalized to the case of multiple receive antennas as well, thus providing receive diversity in addition to transmit diversity. STBC is a technique used in wireless communications to transmit multiple copies of a data stream across a number of antennas and to exploit various received versions of the data to improve reliability of data transfer [2].

The performance of Generalized Selection Combining in a slow and flat Rayleigh fading channel is discussed by many authors as there is now great interest in the system that selects a subset of available antennas for reception. The basics of modulation schemes such as M-ary Phase shift Keying (MPSK) and Quadrature Amplitude modulation (QAM) techniques were discussed [3].

The concept of order statistics is explained in a simplified form [4]. The Alamouti transmission technique [5] is a simple transmit diversity scheme using two transmit antennas and one receive antenna which can be generalized to more than one receive antenna. It is the only existing full rate and full-diversity code for complex signal constellations.

Generalized Selection Combining (GSC) selects \( L_s \) high SNR receiver branches out of all \( L \) diversity branches and combines \( L_s \) selected signals using MRC. Since MRC is sensitive to channel estimation errors, and with GSC, the weak signals which are prone to these errors are excluded in GSC with \( L_s/L \) (selecting \( L_s \) Rx branches out of \( L \) available ones) outperforming a MRC scheme with diversity order \( L_s \) [6].

Space-time block coding introduced generalizes the transmission scheme discovered by Alamouti to an arbitrary number of transmit antennas and is able to achieve full diversity promised by transmit and receive antennas [7]. These codes retain the property of having a very simple maximum likelihood decoding algorithm based on linear processing at the receiver [8]. For real signal constellations (such as PAM), they provide maximum possible transmission rate allowed by the theory of space time coding [9]. For complex constellations, STBCs can be constructed for any number of transmit antennas, and these codes have remarkably simple decoding algorithms based on linear processing at the receiver. They provide full spatial diversity and half of the maximum possible transmission rate is allowed from space-time coding theory.

A GSC scheme based on Log-Likelihood Ratio with a single antenna at the transmitter side is also investigated and numerical results for Bit Error Rate (BER) of Binary Phase Shift Keying are derived. In order to simplify hardware implementation, and reduced power consumption while still retaining good system performance, two new selection methods for using Alamouti MIMO system are proposed [10]. They are the generalisation of the previous methods for BPSK using received signal antenna selection. In general, MIMO antenna Selection Combining (SC) includes includes receiver \((Rx)\) antenna selection, transmitter \((Tx)\) antenna selection, and joint \(Tx/Rx\) selection.
Both $T_x/R_x$ selection and $T_x$ selection require that channel estimation be fed back from the $R_x$ to the $T_x$ [10].

In order to avoid the need for a feedback channel and to keep the system simple, some systems will implement $R_x$ selection diversity only. In MIMO $R_x$ selection diversity, $L_r$ out of $L$ $R_x$ antennas are selected, while the $T_x$ uses all available antennas. It has examined MIMO $R_x$ selection diversity. The $R_x$ selection criteria is based on achieving the maximum received SNR. An approximation of pair wise error probability, an upper bound on pair wise error probability is presented and an upper bound on BER were derived [11].

Five different selection schemes for $R_x$ antenna selection have been considered. The first scheme is Log-Likelihood Ratio (LLR) selection which was proposed for a one $T_x$ antenna and $L$ $R_x$ antennas. In LLR selection, full knowledge of all complex diversity branch gains is needed, and the branch providing the largest magnitude of LLR is chosen. This selection scheme was extended to include two $T_x$ and $N_R$ $R_x$ antennas system using the Alamouti scheme [12].

The BER for this scheme is given by an expression involving a single integral. However, perfect channel estimation is assumed. Here, a closed-form BER expression was derived for this LLR selection scheme accounting for the presence of channel-estimation errors [13].

The second scheme considered is traditional selection combining. The selection of the best antenna is based on the largest SNR among the diversity branches at the detector input. Unlike LLR selection which requires full knowledge of complex channel gains for all diversity branches, SNR selection requires ordering fading amplitudes on the diversity branches. SNR selection is now applied to $T_x$ selection.

Two $T_x$ antennas which provide the largest and the second largest SNR are used for transmitting a STBC. The performance of the system is assessed in terms of outage capacity analysis, but exact BER results are not given. The BER of SNR selection at the $R_x$ side is evaluated and the result is extended to include the effects of channel-estimation errors [14].

The selection combining scheme discussed in this paper are Space Time Sum of Square (STSoS) and Space Time Sum of Magnitude (STSoM). The STSoS SC scheme requires squaring the amplitudes of the received signals. In order to further simplify hardware implementation, another scheme is proposed, which only needs the amplitudes of the received signals. Similar to STSoS selection, STSoM does not require channel estimation. The simulation results in the following section show that STSoM selection has only slightly poorer BER performance than STSoS and SNR selection [15].

Section II involves system model and description of $2 \times 2$ and $4 \times 4$ Alamouti MIMO system using STSoS selection combining scheme. Section III involves BER analysis of $4 \times 4$ and $8 \times 8$ Alamouti MIMO systems. Section IV presents the Numerical results. Finally, Section V presents the conclusions.

2. System Model and its Description

We consider a system where an Alamouti scheme is applied to four transmit and four receive antennas. Four symbol $s_1$, $s_2$ $s_3$ and $s_4$ are transmitted in four consecutive time slots $T_{S_1}$, $T_{S_2}$, $T_{S_3}$ and $T_{S_4}$ using four different antennas at the transmitter as well as the receiver. Let $g_{j,i}$ be the complex gain between the $j^{th}$ transmitting antenna and the $i^{th}$ receiving antenna, where $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$. This system can be shown by Figure 1.
For receiving antenna \(i(1 \leq i \leq 4)\), \(g_{i,i}\) is the direct complex gain coefficient and \(g_{i,j}(i \neq j)\) are the cross coefficients. The corresponding received signal in the four time interval on \(i^{th}\) branch of 4 × 4 Alamouti MIMO system is given by \(r_{1,i}, r_{2,i}, r_{3,i}\) and \(r_{4,i}\) as

\[
\begin{align*}
    r_{1,i} &= g_{1,i} s_1 + g_{2,i} s_2 + g_{3,i} s_3 + g_{4,i} s_4 + n_{1,i}, \\
    r_{2,i} &= g_{1,i} s_2 - g_{2,i} s_1 - g_{3,i} s_4 + g_{4,i} s_3 + n_{2,i}, \\
    r_{3,i} &= g_{1,i} s_3 + g_{2,i} s_4 - g_{3,i} s_1 - g_{4,i} s_2 + n_{3,i} \quad \text{and} \\
    r_{4,i} &= g_{1,i} s_4 - g_{2,i} s_3 - g_{3,i} s_2 - g_{4,i} s_1 + n_{4,i},
\end{align*}
\]

where \(j = 1, 2, 3, 4\) and \(i = 1, 2, 3, 4\).

The Combiner output is given by

\[
\begin{align*}
    y_{1,i} &= \hat{g}_{1,i} r_{1,i} + \hat{g}_{2,i} r_{1,i} + \hat{g}_{3,i} r_{3,i} + \hat{g}_{4,i} r_{4,i}, \\
    y_{2,i} &= \hat{g}_{1,i} r_{2,i} - \hat{g}_{2,i} r_{2,i} + \hat{g}_{3,i} r_{4,i} + \hat{g}_{4,i} r_{3,i}, \\
    y_{3,i} &= \hat{g}_{1,i} r_{3,i} + \hat{g}_{2,i} r_{4,i} - \hat{g}_{3,i} r_{1,i} - \hat{g}_{4,i} r_{2,i}, \quad \text{and} \\
    y_{4,i} &= \hat{g}_{1,i} r_{4,i} + \hat{g}_{2,i} r_{3,i} + \hat{g}_{3,i} r_{1,i} - \hat{g}_{4,i} r_{1,i},
\end{align*}
\]

where \(\hat{g}_{j,i}\) is the estimate of \(g_{j,i}\).

Similarly, the 8 × 8 system can be shown by Figure 2. In this eight symbols \(s_1, s_2, \ldots, s_8\) are transmitted in eight consecutive time slots \(TS_1, TS_2, \ldots, TS_8\) using eight different antennas at the transmitter as well as the receiver. Let \(g_{j,i}\) be the complex gain between the \(j^{th}\) transmit antenna and the \(i^{th}\) receive antenna, where \(i = 1, 2, \ldots, 8\) and \(j = 1, 2, \ldots, 8\). For receive antenna \(1\), \(g_{1,1}\) is the direct complex gain coefficient and \(g_{2,1}, g_{3,1}, \ldots, g_{8,1}\) are the cross coefficients. For receive antenna 2, \(g_{2,2}\) is the direct complex gain coefficient and \(g_{1,2}, g_{3,2}, \ldots, g_{8,2}\) are the
BER Comparison of $4 \times 4$ and $8 \times 8$ Alamouti MIMO Systems

Cross coefficients. Similarly for receive antenna 8, $g_{88}$ is the direct complex gain coefficient and $g_{18}, g_{28}, \ldots, g_{78}$ are the cross coefficients. The corresponding received signal in the eight time interval on the $i$th branch of $8 \times 8$ Alamouti MIMO system is given by $r_{1i}, r_{2i}, \ldots, r_{8i}$, where

$$
r_{1i} = g_{1i}s_1 + g_{2i}s_2 + g_{3i}s_3 + g_{4i}s_4 + g_{5i}s_5 + g_{6i}s_6 + g_{7i}s_7 + g_{8i}s_8 + n_{1i},
$$

$$
r_{2i} = g_{2i}s_1 - g_{1i}s_2 + g_{5i}s_5 - g_{4i}s_4 + g_{3i}s_3 - g_{6i}s_6 + g_{7i}s_7 + g_{8i}s_8 + n_{2i},
$$

$$
\vdots
$$

$$
r_{8i} = g_{8i}s_1 - g_{7i}s_2 - g_{6i}s_3 - g_{5i}s_4 + g_{4i}s_5 + g_{3i}s_3 + g_{2i}s_7 - g_{2i}s_8 + n_{8i},
$$

where $j = 1, 2, \ldots, 8$ and, $i = 1, 2, \ldots, 8$.

The Combiner output is given by

$$
y_{1i} = \hat{g}_{1i}r_{1i} + \hat{g}_{2i}r_{2i} + \hat{g}_{3i}r_{3i} + \hat{g}_{4i}r_{4i} + \hat{g}_{5i}r_{5i} + \hat{g}_{6i}r_{6i} + \hat{g}_{7i}r_{7i} + \hat{g}_{8i}r_{8i} + n_{1i},
$$

$$
y_{2i} = \hat{g}_{2i}r_{1i} - \hat{g}_{1i}r_{2i} + \hat{g}_{5i}r_{5i} - \hat{g}_{4i}r_{4i} + \hat{g}_{3i}r_{3i} - \hat{g}_{6i}r_{6i} + \hat{g}_{7i}r_{7i} + \hat{g}_{8i}r_{8i} + n_{2i},
$$

$$
\vdots
$$

$$
y_{8i} = \hat{g}_{8i}r_{1i} - \hat{g}_{7i}r_{2i} - \hat{g}_{6i}r_{3i} - \hat{g}_{5i}r_{4i} + \hat{g}_{4i}r_{5i} + \hat{g}_{3i}r_{6i} + \hat{g}_{2i}r_{7i} - \hat{g}_{1i}r_{8i} + n_{8i},
$$

3. Bit-Error Rate Analysis

Combining (1), (2), (3), (4) and (8), the combiner’s output $y_{1i}$ can be written as

$$
y_{1i} = k(|\hat{g}_{1i}|^2 + |\hat{g}_{2i}|^2)s_1 + (\hat{g}_{1i}^*d_{1,i} + \hat{g}_{2i}d_{2,i}^* + \hat{g}_{2i}d_{2,i}^* + \hat{g}_{3i}^*d_{3,i} + \hat{g}_{4i}d_{4,i} + \hat{g}_{5i}d_{5,i} + \hat{g}_{6i}d_{6,i} + \hat{g}_{7i}d_{7,i}^* + \hat{g}_{8i}d_{8,i}^*)s_2 + (\hat{g}_{1i}^*n_{1,i} + \hat{g}_{2i}n_{2,i}^*).
$$

Since $s_2 = +s_1$ or $-s_1$ each with probability 1/2, BER can be calculated as

$$
P_e = \frac{1}{2}(P_{e,s_2=s_1} + P_{e,s_2=-s_1}).
$$

35
The Probability of error is given as

\[ P_e = p^N \sum_{k=0}^{N-1} (N-1+k) C_k (1-p)^k, \quad (13) \]

where

\[ p = \frac{1}{2} - \frac{1}{2} \left( 1 + \frac{1}{E_b/N_0} \right)^{-1/2} \quad (14) \]

is the cross over probability, \( E_b/N_0 \) is the ratio of the energy per bit to noise spectral density. Here, \( N \) is the number of the bits transmitted. In this case, \( N = 2 \). So, \( P_e \) can be rewritten as

\[ P_e = p^2 (1 + 2(1 - p)). \quad (15) \]

4. Numerical Results

In this section, BER performance for different number of transmit and receive antennas is compared using BPSK modulation with Alamouti STBC scheme for a Rayleigh fading channel.

Figure 3 shows BER versus SNR \( (E_b/N_0) \) curves for 1 x 1 (Tx = 1, Rx = 1), 2 x 2 (Tx = 2, Rx = 2) and 4 x 4 (Tx = 4, Rx = 4) systems. It is inferred that the 4 x 4 system gives better BER performance than 2 x 2 and 1 x 1 systems as 4 x 4 system is having BER below \( 10^{-1} \).

Figure 4 shows BER versus SNR \( (E_b/N_0) \) curves for different number of Tx antennas with Rx number of antenna = 4. In all cases, the number of Tx antennas is greater than or equal to the Rx antennas. It can be observed that as the number of Tx antennas increases, the probability of error increases, and BER performance improves.

Figure 5 shows BER versus SNR \( (E_b/N_0) \) curves for 2 x 2 (Tx = 2, Rx = 2), 4 x 4 (Tx = 4, Rx = 4) systems and 8 x 8 (Tx = 8, Rx = 8) systems. It can be observed that a 8 x 8 system has

![Figure 3. BER using BPSK modulation with Alamouti STBC (Rayleigh Fading).](image-url)
Figure 4. BER using BPSK modulation with Alamouti STBC (Rayleigh Fading) for 4 × 4, 5 × 4 and 6 × 4.

Figure 5. BER comparison using BPSK modulation with Alamouti STBC for 2 × 2, 4 × 4 and 8 × 8 systems.

improved BER performance compared to a 2 × 2 and 4 × 4 systems because 8 × 8 system has better reception as number of transmit and receive antennas are higher.

Figure 6 shows BER versus SNR ($E_b/N_0$) curves for different number of Rx antennas when the number of Tx antennas = 8. It can be observed that as the number of receive antennas is greater than the number of transmit antennas, probability of error decreases which improves BER performance of the system.
Figure 6. BER comparison using BPSK modulation with Alamouti STBC for $8 \times 6$, $8 \times 7$ and $8 \times 8$ systems.

Figure 7. BER comparison using BPSK modulation with Alamouti STBC for $8 \times 8$, $9 \times 8$ and $10 \times 8$ systems.

Figure 7 shows BER versus SNR ($E_b/N_0$) curves for different number of $Tx$ with $Rx = 8$. It can be observed that as the number of transmit antennas is greater than the number of receive antennas, probability of error increases which degrades BER of the system.

5. Conclusion

In this paper, analytical BER results were derived for selection combining with channel estimation errors using Alamouti transmission system. A new selection scheme named STSoS scheme was proposed with a still simpler implementation. The results demonstrated that system performance improves with increase in the number of transmit and receive antennas. It is observed that BER
BER Comparison of 4 × 4 and 8 × 8 Alamouti MIMO Systems

The performance of a 8 × 8 Alamouti MIMO system is better than a 4 × 4 Alamouti MIMO system. The effect of channel estimation errors on BER performance for selection combining was illustrated and discussed.

References