Alpha Weighted Quadratic Filter Based Enhancement for Mammogram

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Abstract. Mammograms are widely used to detect breast cancer in women. The quality of the image may suffer from poor resolution due to the limitations of the X-ray hardware systems. Image enhancement is a powerful tool to improve the visual quality of mammograms. This work presents a new nonlinear alpha weighted quadratic filter employing enhancement technique in image processing. The algorithm is based on masking technique, which can remove noise efficiently from highly corrupted gray scale images while preserving image details. LogAMEE is used for enhancement measure and experimental results shows that AWQF provides an excellent enhancement results on existing techniques.

Keywords: Enhancement, AWQF, LogAMEE.

1. Introduction

Breast cancer is the leading cause of death in women between the ages of 35 and 55. Breast cancer is a disease where abnormal cells grow in an uncontrolled fashion. The National Cancer Institute estimates that one out of eight women in the United States will develop breast cancer at some point during her lifetime [1]. Early detection of breast cancer is an important and effective method to reduce mortality since treatments of breast cancer in the early stages are more successful. Mammography is the most common technique used by radiologists in the screening and diagnosis of breast cancer. It is seen as the most reliable method for early detection of breast carcinomas [2]. X-ray mammography is the most common technique used by radiologists in the screening and diagnosis of breast cancer. Although it is seen as the most reliable method for early detection of breast carcinomas, reducing mortality rates by up to 25%, its interpretation is very difficult—10%–30% of breast lesions are missed during routine screening. To increase the diagnostic performance of radiologists, several computer-aided diagnosis schemes have been developed to improve the detection of either of the two primary signatures of this disease: 1) masses and 2) micro calcifications. Apart from this two more abnormalities are asymmetry between images of left and right breasts and distortion of the normal architecture of breast tissue.

Image enhancement is basically improving the interpretability or perception of information in images for human viewers and providing ‘better’ input for other automated image processing techniques. The principal objective of image enhancement is to modify attributes of an image to make it more suitable for a given task and a specific observer. During this process, one or more attributes of the image are modified. The choice of attributes and the way they are modified are specific to a given task. Moreover, observer-specific factors, such as the human visual system and the observer’s experience, will introduce a great deal of subjectivity into the choice of image enhancement methods. There exist many techniques that can enhance a digital image without spoiling it.

The paper is organized as follows: Section 2 describe system design, section 3 presents implementation details, section 4 describe the experimental results and evaluates the performance of the proposed system. Finally the conclusion from this work is summarized in section 5.

2. Problem Solution

In the present work a new nonlinear alpha weighted quadratic filter employing enhancement technique in image processing. The algorithm is based on masking technique, which can remove noise efficiently from highly corrupted...
gray scale images while preserving image details. Due to its complexity and it requires a large number of coefficients to be calculated. The users have difficulty designing an AWQF for practical applications because all its coefficients have to be determined. On the other hand, more coefficients offer the AWQF more power and more design flexibility to meet more specific and complex requirements in real world applications. Therefore, it can be used to remove the noise and enhance in many medical applications for its performance.

To make the AWQF independent of the orientation of the objects or features of the input image, the AWQF is designed into an isotropic image operator. Thus, the number of the AWQF’s coefficients can be reduced. Here enhancement process mainly consists of firstly designing the filter and then applying designed filter to the mammographic images.

3. System Design

This work is organized as follows:

Input image - The images are taken from MIAS database.

Image enhancement technique - To improve the visual quality of mammograms by using the Alpha weighted quadratic filter (AWQF).

Enhanced image - We will get the high resolution and high contrast images as the output.

A. Quadratic filter (Volterra Filter)

In order to generalize nonlinear system theory, the Volterra series have been recognized as a powerful mathematical tool leading to another class of nonlinear filters called polynomial filters [11]. Volterra series expansions represent an important model for the representation, analysis and synthesis of nonlinear systems. However, a significant problem with this approach to filter design is that the number of terms required to estimate grows exponentially with the order of expansion. In practice, therefore, we truncate the Volterra series to consist of, at most, second degree terms only. Quadratic filters are generalization of linear filters in which the filter output contains a linear combination of products of the input time series. Volterra filters are approximately equivalent to the product of a local mean estimator and a generalized high pass filter. Volterra filters are proposed for edge preserving image restoration using block lexicographic matrices and symmetry conditions.

A class of nonlinear 2D filter is designed based on the truncated discrete Volterra series together with a matrix notation [12]. Exponential nonlinear Volterra filter is used for contrast sharpening in noisy images and the design of filter is based on the theory of Generalized Fock spaces (GF) of Volterra series. Volterra filters are used for extraction and enhancement of edges in images also. A class of nonlinear 2D filter is designed based on the truncated discrete Volterra series together with a matrix notation [11]. Exponential nonlinear Volterra filter is used for contrast sharpening in noisy images and the design of filter is based on the theory of Generalized Fock spaces (GF) of Volterra series [13]. Volterra filters are used for extraction and enhancement of edges in images also.

This method has following steps.

Step 1: For the given image find the average intensity to select the appropriate cut-off frequency.
Step 2: Obtain the noisy image by including additive Gaussian and mixed Gaussian-impulse noise.
Step 3: Calculate the linear and nonlinear filter coefficients using FIR technique with suitable windowing method.
Step 4: Reduce the number of coefficients using Block Lexicographic matrix and symmetry conditions.
Step 5: Convolve the input noisy image with the filter coefficients to obtain the filtered image.
Step 6: Evaluate the performance of the filter using different quantitative parameters.

Its 1D format is characterized by the following equation:

\[ y(n) = \sum_{j,k=-M}^{M} w(j, k)x(n - j)x(n - k) \]
where \( N = 2M + 1 \) is the mask size and \( w(i) \) is coefficients. Similar to the polynomial filter, the quadratic filter is also known to be a complex nonlinear filter [4]. Its implementation requires a large number of coefficients to be determined in the design of the filter.

Assuming the mask size is \( N \times N \), \( N = 2M + 1 \), the 2D format of quadratic filter is defined as,

\[
y(m, n) = \sum_{i,j} w(i, j) x(m-i, n-j) x(m-k, n-l) \\
or \quad y_n = \sum_{j,k} w_j x_j x_k
\]

### B. Alpha weighted quadratic filter

The Alpha Weighted Quadratic Filter (AWQF) is an extension of the quadratic filter which is defined as following,

\[
y(n) = \sum_{i,j} w(i, j) x^{a(i)}(n-i) x^{a(j)}(n-j)
\]

where \( w(i) \) and \( a(i) \) are coefficients. Similarly, its 2D format is defined as following,

\[
y(m, n) = \sum_{i,j} w(i, j, k, l) x^{a(i, j)}(m-i, n-j) x^{a(k, l)}(m-k, n-l) \\
or \quad y_n = \sum_{j,k} w_j x_j x_k
\]

<table>
<thead>
<tr>
<th>( w_{1a_1} )</th>
<th>( w_{2a_2} )</th>
<th>( w_{3a_3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_{4a_4} )</td>
<td>( w_{5a_5} )</td>
<td>( w_{6a_6} )</td>
</tr>
<tr>
<td>( w_{7a_7} )</td>
<td>( w_{8a_8} )</td>
<td>( w_{9a_9} )</td>
</tr>
</tbody>
</table>

For example, if mask size is \( 3 \times 3 \), \( N = 3 \) (i.e., = 1), the mask contains nine image pixels. According to the distance between two elements of the AWQF which are two pixels of the input image, the AWQF can be classified into three types:

a) **Type Zero AWQF**: This type of the AWQF consists of all the second order terms in which the distance of two elements is zero, namely two elements have the same locations. For mask size \( 3 \times 3 \), the filter is,

\[
y_n = w_{55} x_5^{2a_5} + w_{11}(x_2^{a_1} + x_3^{a_2} + x_7^{a_7}) + w_{22}(x_2^{a_2} + x_4^{a_4} + x_8^{a_8})
\]

b) **Type One AWQF**: The type one AWQF includes all the second order terms in which the distance of two elements is one. In other words, two elements are two adjacent pixels. For mask size \( 3 \times 3 \), the filter becomes

\[
y_n = w_{12}(x_1^{a_1} + x_2^{a_2} + x_3^{a_3}) + w_{15}(x_1^{a_1} x_5^{a_5} + x_2^{a_2} x_5^{a_5} + x_3^{a_3} x_5^{a_5}) + w_{24}(x_2^{a_2} + x_4^{a_4} + x_8^{a_8})
\]

### 4. Implementation

This section gives the steps carried out in the implementation of the system. Here we have used the MATLAB tool to carry out the processing task. The step by step implementation process carried out is represented in the following sections.
A. AWQF implementation algorithm

There are a large number of coefficients in the AWQF. This makes the AWQF quite expensive to implement. For example, there are 99 coefficients in the AWQF if mask size is $3 \times 3$. Seventeen coefficients still remain even if we use symmetric and isotropic properties to reduce the number of its coefficients. For a larger mask size, the number of coefficients will significantly increase. In this section, we introduce a new algorithm to simplify the AWQF implementation for a large mask size. The algorithm is described as follow:

1) If mask size is $3 \times 3$, the input image is directly filtered by the AWQF.
2) If mask size is $5 \times 5$, we select 9 image pixels from the $5 \times 5$ mask window to generate a new $3 \times 3$ window and then apply the AWQF.
3) If the mask size is greater than $5 \times 5$, we apply the AWQF to the $3 \times 3$ sub-windows in the four corners of the mask window. The final output of the AWQF is the mean value of the results from these four sub-windows.

c) Type Two AWQF: It is comprised of the second order terms in which the distance of two elements is two. For mask size $3 \times 3$, the filter is,

$$
\begin{align*}
    y_n &= w_{13}(x_1 a_1^1 + x_3 a_1^1 + x_1 a_2^2 + x_7 a_2^2 + x_2 a_1^1 + x_9 a_1^1 + a_1 a_1^1) + w_{19}(x_1 a_1 + x_3 a_1 + a_1 a_1) + w_{28}(x_2 a_2 + x_4 a_2^2 a_2) \\
    &\quad + w_{16}(x_1 a_6^1 + x_1 a_8^2 + x_2 a_7 + x_2 a_2 + x_8 a_2 + x_9 a_2 + x_2 a_2 + x_3 a_2 + a_2 a_2 a_1)
\end{align*}
$$

B. Performance measure

To quantitatively evaluate the AWQF’s enhancement performance, the user has the flexibility to use any measure approach for establishing a qualitative metric of mammogram enhancement. They can also optimize the AWQF’s coefficients using the measure results to obtain better enhanced mammograms. In the present work, we utilize the logarithmic Michelson contrast measure by entropy (LogAMEE) [14] as an example of the enhancement measure methods to design the AWQF.

To further simplify implementation and reduce the number of the coefficients of the AWQF, we assume

$$
\begin{align*}
    w_1 = w_2 = h, & a_1 = a_2 = h, & w_{15} = w_{25} = w_{16} = h, & w_{12} = 4h, \\
    w_{11} = w_{22} = -h & & & -1 < h < 1 \\
    w_{24} = 2w_{12} = -h \\
    w_{19} = 2w_{13} = -4h, & w_5 = a_5 = 1 - 8h, & w_{55} = 8h
\end{align*}
$$

An important step in a direct image enhancement approach is to create a suitable performance measure. The improvement found in the resulting images after enhancement is often very difficult to measure. This problem becomes more apparent when the enhancement algorithms are parametric, and one needs to choose the best parameters, to choose the best transform among a class of unitary transforms, or to automate the image enhancement procedures. The problem becomes especially difficult when an image enhancement procedure is used as a preprocessing step for other image processing purposes such as object detection, classification, and recognition.

Many enhancement techniques are based on enhancing the contrast of an image [15–18]. There have been many differing definitions of an adequate measure of performance based on contrast [19–21]. Gordon and Rangayan used local contrast defined by the mean gray values in two rectangular windows centered on a current pixel. Beghdadi
and Negrate [19] defined an improved version of the aforementioned measure by basing their method on a local edge information of the image. Measures of enhancement based on the HVS have been in the present work. Two definitions of contrast measure have traditionally been used for simple patterns [22]: Michelson [23,24], which is used for periodic patterns like sinusoidal gratings, and Weber, which is used for large uniform luminance backgrounds with small test targets [22]. However, these measures are not effective when applied to more difficult scenarios such as images with non uniform lighting or shadows [22]. The first practical use of a Weber’s-law based contrast measure, the image enhancement measure or contrast measure, was developed by Agaian [25].

This contrast measure was later developed into the EME or measure of enhancement, and the EMEE, or measure of enhancement by entropy [26]. Finally, the Michelson Contrast Law was included to further improve the measures. These were called the AME and AMEE [26]. These are summarized in [26] and are calculated by dividing an image into \( k \times k \) blocks, by calculating the measure for each block, and by averaging the results as shown in the formula definitions.

In the present work we capitalize on the strengths of these measures and improve upon them by including the PLIP operator primitives. This allows for a more accurate processing of the contrast information, as PLIP subtraction has definitions.

**C. Plip model and arithmetic**

The PLIP model was introduced by Panetta et al. [?] to provide a nonlinear framework for image processing, which addresses these five requirements. It is designed to both maintain the pixel values inside the range \([0,M]\) and more accurately process images from an HVS point of view. To accomplish this, images are processed as absorption filters using the gray tone function. This gray tone function is as follows:

\[
g(i, j) = M - f(i, j)
\]

where \(f(i, j)\) is the original image function, \(g(i, j)\) is the output gray tone function, and \(M\) is the maximum value of the range. It can be seen that this gray tone function is much like a photo negative.

\[
a \oplus b = a + b - \frac{ab}{\gamma(M)}
\]

\[
a \otimes b = k(M) \frac{a - b}{k(M) - b}
\]

\[
c \otimes a = \gamma(M) - \gamma(M) \left(1 - \frac{a}{\gamma(M)}\right)^c
\]

\[
a \ast b = \varphi^{-1}(\varphi(a) \cdot \varphi(b))
\]

\[
\varphi(a) = -\lambda(M) \cdot \ln^\beta \left(1 - \frac{f}{\lambda(M)}\right)
\]
Table 1. Comparison of LogAMEE value of type 0, type 1, type 2 AWQF with histogram equalization when $h = 0.1$.

<table>
<thead>
<tr>
<th>Image name</th>
<th>LogAMEE of Type 0 AWQF</th>
<th>LogAMEE of Type 1 AWQF</th>
<th>LogAMEE of Type 2 AWQF</th>
<th>Histogram Equalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Img 1</td>
<td>268</td>
<td>276</td>
<td>343</td>
<td>41.7</td>
</tr>
<tr>
<td>Img 2</td>
<td>171</td>
<td>220</td>
<td>318</td>
<td>91.1</td>
</tr>
<tr>
<td>Img 3</td>
<td>82.7</td>
<td>97.1</td>
<td>109</td>
<td>46.9</td>
</tr>
<tr>
<td>Img 4</td>
<td>94.9</td>
<td>100</td>
<td>141</td>
<td>62.5</td>
</tr>
<tr>
<td>Img 5</td>
<td>107</td>
<td>218</td>
<td>247</td>
<td>84.9</td>
</tr>
</tbody>
</table>

Figure 2. LogAMEE for type 0, type 1, type 2 AWQF.

Figure 3. LogAMEE for images of type 0, type 1, type 2 AWQF when $h = 0.1$ and histogram equalization.

5. Experiments and Results

In this section, the comparison of type 0, type 1 and type 2 AWQF is carried out. The performance of proposed AWQF is compared with other image enhancement technique such as, histogram equalization. To measure the performance numerically, logarithmic measure of enhancement by entropy of the enhanced image with original image is evaluated.

Both the simulation results and computational complexity analysis show that the proposed method is better than many of the existing enhancement techniques.

6. Conclusion

This work presents a designing of alpha weighted quadratic filter which is an extension of quadratic filter which is used to enhance the images. AWQF could remove noise while preserving image details and textures very well and effectively enhance the overall contrast of mammograms and also improve the local fine details and dark regions in mammograms. Automatic segmentation is also achieved by using AWQF. Compared to the histogram equalization method, the AWQF shows better performance for enhancing mammograms. A numerical measure, such as LogAMEE and visual observation shows the convincing results.

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References


