Finite Element Method for Designing and Analysis of the Transformer – A Retrospective

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Abstract—Finite Element Analysis (FEA) using Finite Element Method (FEM) was developed over 70 years to solve the complex elasticity and structural analysis problem in civil and aeronautical engineering. Application of FEA is being expanded to simulation in electrical engineering also to solve the complex design problems. The circuit theory models for designing transformers are not much accurate in determining the transformer parameters such as winding impedance, leakage inductance, hot spot temperature etc. The physical realization of these parameters is needed on a prototype unit. The finite element method can play a vital role in deriving these parameters without any physical verification. An effort has been made in this paper to show the effectiveness of finite element method in determining the above said parameters while designing the transformers - both oil cooled as well as dry type - for power and distribution sectors as well as to analyze and detect the internal faults in the transformer.

Index Terms—Finite Element Method, Finite Element Analysis, Quasi-static Finite Element, Leakage Inductance,

I. INTRODUCTION

The finite element method (FEM) rapidly grew as the most useful numerical analysis tool for engineers and applied mathematicians because of its natural benefits over prior approaches. The main advantages of FEM are, it can be applied to arbitrary shapes in any number of dimensions, the material properties can be non-homogeneous (depend on location) and/or anisotropic (depend on direction). The way that the shape is supported (also called fixtures or restraints) can be quite general, as can be the applied sources (forces, pressures, heat, flux, etc.). The FEM provides a standard process for converting the governing energy principles or governing differential equations in to a system of matrix equations to be solved for an approximate solution. For linear problems such solutions can be very accurate and quickly obtainable.

The objective of applying finite element method while designing a transformer is, to estimate the temperature rise and hot spots under non linear load conditions, to determine the impedance value and short circuit forces to identify the electrical stresses and weakest insulation points and to estimate the correct insulation level to avoid the HV failures.

This paper discusses the use and effectiveness of finite element method in designing the power transformer as compared to magnetic circuit theory [12], internal winding fault detection and analysis [3, 4], a three dimensional finite element analysis of electric fields at winding ends of dry type transformer [9], hot spot and life evaluation of power transformer [6, 13], leakage inductance calculations [7] and transformer over heating under non linear load conditions [8]

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II. Transformer Design Using FEM

In designing a transformer, especially power transformers, parameters like impedance of the transformer, temperature rise and hot spot evaluation, rated short time overloading capacity and adequate insulation level are of prime importance. Determination of these parameters at design stage has a very big impact on the cost effectiveness of the transformer.

In a paper presented by S C Bell and S Bodger [2, 15], a comparison has been made between magnetic circuit theory and finite element method for designing the power transformer. It summarizes the reverse method of transformer design.

In this paper, first the models for the resistive and inductive reactance components of the Steinmetz ‘exact’ transformer equivalent circuit have been developed from fundamental theory as presented in [13]. Then two and three dimensional linear and non linear magneto static finite element models were introduced as an alternative model for the inductive reactance components.

To demonstrate the reverse design method, the author have designed, built and tested two single phase, 50 Hz, high voltage transformers, the results of which are summarized in table I.

<table>
<thead>
<tr>
<th>Method</th>
<th>Equivalent Circuit Parameters in Ω</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_C$</td>
<td>$R_W$</td>
<td>$X_M$</td>
<td>$X_I$</td>
</tr>
<tr>
<td>Transformer 1</td>
<td>Primary 240 V, Secondary 6240 V, 200 VA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DM</td>
<td>3388</td>
<td>10.0</td>
<td>1987</td>
<td>2.8</td>
</tr>
<tr>
<td>CTM</td>
<td>1342</td>
<td>11.5</td>
<td>1383</td>
<td>1.9</td>
</tr>
<tr>
<td>LFEM</td>
<td>-</td>
<td>-</td>
<td>1905</td>
<td>1.6</td>
</tr>
<tr>
<td>NLFEM</td>
<td>-</td>
<td>-</td>
<td>1883</td>
<td>-</td>
</tr>
<tr>
<td>Transformer 2</td>
<td>Primary 14 V, Secondary 4560 V, 617 VA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DM</td>
<td>18</td>
<td>0.043</td>
<td>41</td>
<td>0.012</td>
</tr>
<tr>
<td>CTM</td>
<td>9.9</td>
<td>0.055</td>
<td>20</td>
<td>0.016</td>
</tr>
<tr>
<td>LFEM</td>
<td>-</td>
<td>-</td>
<td>25</td>
<td>0.015</td>
</tr>
<tr>
<td>NLFEM</td>
<td>-</td>
<td>-</td>
<td>54</td>
<td>-</td>
</tr>
</tbody>
</table>

$R_C$ = Transformer core resistance  
$R_W$ = Transformer winding resistance  
$X_M$ = Magnetizing reactance of the transformer  
$X_I$ = Leakage reactance of the transformer  
DM = Direct Measurement  
CTM = Circuit theory model  
LFEM = Linear finite element model  
NLFEM = Non-linear finite element model

The results show that the non linear finite element model most accurately calculated the magnetizing reactance value of the two sample transformers. For transformer 1 the non linear model seems to be less accurate however this may be due to the approximations made in the geometry of the finite element model.

D. Azizian, M. Vakilian, and J. Faiz [7] have introduced analytical and FEM based models for electromagnetic modeling and inductance calculation in dry type multi-winding traction transformers. The accuracy of these models is verified against the experimental data for a traction transformer having specifications as given in table II.

The test results are as shown in table III from which it is evident that the FEM has better and most accurate results. The accuracy of axi-symmetric 2D FEM model goes on increasing as we increase the complexity of modeling from simplest core (2D-S) to full core (2D-F) to improved core (2D-I). The accuracy of 3D model is much higher than the 2D models.

The experimental results of this paper also yield that the proper modeling of the transformer core further helps in determining the value of leakage inductance due to change in the cross section area of the core. This is especially very helpful in designing the converter duty transformers where leakage inductance between the transformer windings is of utmost importance to decide the capacitance value for the LCL filter. The leakage inductance of the transformer itself forms a part of the filter circuit.
### Table II. Specifications Of Typical Traction Transformer

<table>
<thead>
<tr>
<th>Power Rating</th>
<th>Vector Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000 KVA</td>
<td>Dd0/y11</td>
</tr>
<tr>
<td>HV Line Voltage</td>
<td>HV Line Current</td>
</tr>
<tr>
<td>(H1</td>
<td></td>
</tr>
<tr>
<td>LV Line Voltage</td>
<td>LV Line Current</td>
</tr>
<tr>
<td>(L1</td>
<td></td>
</tr>
</tbody>
</table>

### Table III. Leakage Inductances Of Traction Transformer

<table>
<thead>
<tr>
<th>Test / Measurement Method</th>
<th>L1-H1</th>
<th>L1-H2</th>
<th>L1-L2</th>
<th>L2-H1</th>
<th>L2-H2</th>
<th>H1-H2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical Results</td>
<td>5.9</td>
<td>72.5</td>
<td>55.2</td>
<td>71.7</td>
<td>88.1</td>
<td>6.0</td>
</tr>
<tr>
<td>2D-Simplest core</td>
<td>5.679</td>
<td>44</td>
<td>45</td>
<td>44</td>
<td>42</td>
<td>5.686</td>
</tr>
<tr>
<td>2D-Full Core</td>
<td>5.808</td>
<td>111</td>
<td>112</td>
<td>112</td>
<td>110</td>
<td>5.84</td>
</tr>
<tr>
<td>2D-Improved Core</td>
<td>5.794</td>
<td>98</td>
<td>100</td>
<td>99</td>
<td>97</td>
<td>5.834</td>
</tr>
<tr>
<td>3D Model</td>
<td>5.892</td>
<td>88</td>
<td>88</td>
<td>88</td>
<td>88</td>
<td>5.914</td>
</tr>
<tr>
<td>Experimental</td>
<td>5.797</td>
<td>74</td>
<td>73</td>
<td>72</td>
<td>72</td>
<td>5.943</td>
</tr>
</tbody>
</table>

### III. LIFE EVALUATION OF TRANSFORMER USING FEM

One of the most important parameters governing the life expectancy of a transformer is the hot-spot temperature value. Stray losses and non linear loads in a transformer are one of the main contributing factors in creating such hot spots which all together decide the life of a transformer.

The stray losses in a transformer are caused by the time variable leakage flux which induces emf and circulates eddy currents in the winding as well as conducting parts of the transformers such as clamps, core, tank wall etc. Evaluation of stray losses can be done quite accurately by FEM as discussed by A. S. Reddy and M. Vijaykumar [6].

Dejan Susa [13] has developed the models to determine the hottest spot in a transformer based on heat transfer theory, application of the lumped capacitance method, the thermal-electrical analogy and a new definition of nonlinear thermal resistances at different locations within a power transformer. The changes in oil viscosity, loss variation with temperature and changes in transformer time constants due to changes in oil viscosity were also accounted for in the thermal models. The results showed that the top oil temperature time constant is shorter than the time constant suggested by the present IEC loading guide, especially in cases where the oil is guided through the windings in a zigzag pattern for the ONAN and ONAF cooling modes. The models are validated using experimental results, which have been obtained from a series of thermal tests performed on a range of power transformers.

With development of the electronic equipments such as Computers, UPS, and High Frequency Drives for motor loads, arc furnaces, Electronic Ballasts, Compact Fluorescent Lamps, etc. the harmonic content in the power distribution network has increased tremendously. These highly non linear loads are present not only in the industrial sectors but also in the commercial sector and the first victim to any such load is always a transformer feeding them. These non linear loads result into the overheating of the transformers. The non linearity of the loads is best judged by the K factor [1] as described by equation (1) and accordingly the transformer required will be designated as K-rated transformer.

The K-rated transformer does not mitigate the harmonic but is capable enough to sustain the overheating due to such non linear loads. The design and application considerations along with testing approach based on UL, NEMA and IEEE standard C57.110-1986 are best described by L. W. Pierce, member IEEE [5].

\[
K = \sum_{h=1}^{n} \left( \frac{i_h^2}{I_{rms}} \right) h^2
\]  

(1)

Where,

\[ h = \text{order of harmonic} \]
\[ I_h = \text{RMS value of the current for h order harmonic} \]
\[ I_{rms} = \text{RMS value of the total load current} \]

A three dimensional finite element method using a magnetic scalar potential formulation is applied [8] to compute the magnetic field in free and iron spaces. The calculation is then combined with a mixed analytical and numerical form of the electrical circuit equation to take into account the skin and proximity effects in the rectangular windings in dry type transformers under non linear load conditions. A two step FEM using the both reduced and total magnetic scalar potentials throughout a penalty term [10] is proposed in [8]. Accordingly the magnetic field into the air gap (\( \Omega_a \)) and in the core (\( \Omega_c \)) based on the reduced and total magnetic scalar potential \( \phi \) follows the governing equations:

\[ \ln(\Omega_a)H = H_j - \text{grad}(\Phi) \quad \text{and} \quad \ln(\Omega_c)H = -\text{grad}(\Phi) \]

Where, the source magnetic field \( H_j \) can be computed using either the Biot-Savart law or a fictitious field for the known current distribution [11]. The relationship given in equation (2) has been solved from a two step formulation using a difference magnetic field \( h = H - H_0 \), considered as being a disturbance of \( H_0 \) due to the core saturation [10]. In first step the magnetic permeability \( \mu \) is assumed to be infinite and a reduced magnetic scalar potential \( \Phi_1 \) is used to calculate the magnetic field \( H_0 \). In the second step, a total magnetic scalar potential \( \Phi_2 \) formulation is applied with \( H_0 \) as the new magnetic source field in order to calculate the difference field \( h \).

Based on the magnetic field described above and the empirical formula of the ac resistance component as in [14], the additional losses in the winding due to the harmonic currents have been modeled in [8] on a sample 10 KVA dry type distribution transformer. The nominal current of the transformer is applied to the windings for all frequencies in the above paper. However with non linear loads the harmonics do not own the same amplitude. The power losses are then need to be weighted and modeled in accordance with the current to estimate the exact value of temperature rise.

The other major factor that decides the life of a transformer is the designing of insulation system. The determination of insulation level can best be identified by determining the electric fields at the winding ends. Using three dimensional FEM and ANSYS software, J. Hong, L Heyun and X. Zihong [9] have estimated the electric fields at the winding end in 10KV SG10 dry type transformer. The similar approach can be extended to other higher rating transformer too at the design stage itself to estimate the correct insulation level and consequently increase the life of a transformer.

IV. FAULT DETECTION IN TRANSFORMER USING FEM

Majority times the faults in a transformer are turn to turn, disk to disk or turn/disk to earth short circuit. This occurs mainly because of the aging of the insulation or displacement of the insulation during transportation or maintenance. This leads to the overheating of the transformer and finally results into the transformer failure.

Study shows that around 70-80% of the transformer failures are due to the short circuit between turns. Considering this damage occurring in the transformer and time and cost involved in rectifying the same, it seems that simulation involving modeling of the transformer for detecting the fault is the most economical and convenient way.

One of such methods of modeling distribution transformer with internal short circuit faults using Finite Element Analysis (FEA) is presented in [4]. Based on the physical information of the transformer, the finite element model for a normal transformer or a transformer with an internal fault was implemented by commercially available software. The resulting circuit model was exported and used in a circuit analysis package to study the terminal behavior of the transformer. The observations of the simulation are compared with the experimental results and it shows that the FEA model can provide an accurate estimation of the internal winding faults.

The electromagnetic quasi-static finite element approach [3] describes the detection of winding short circuit faults with the help of frequency response analysis. The principle of coupled circuit approach and electromagnetic quasi-static analysis approach were applied along with FEA for determining the short circuit current under different fault conditions.

A case study on 30 MVA, 63 KV/20 KV, Ynd1, and 50 Hz transformer has been discussed in this paper [3]. Parameters of transformer are estimated by means of finite element analysis and utilized in circuit based
model and the input impedance is calculated in wide band frequency. In addition to classifying and analyzing main types of short circuit according to IEEE standard C57.140, frequency response of disk-disk short circuit state is also investigated in several points in the HV winding to identify the location of the short circuit fault along the winding. The results have been summarized in terms of deviation of resonance component which showed that the first resonance of the input impedance due to short circuit moves orderly giving a better idea about the location of the short in a winding.

V. CONCLUSION

The discussion above reveals that the finite element method is an efficient tool for designing the transformer. Whereas the determination of impedance, winding hot spots temperature, insulation class and insulation level etc. are the major challenges while designing a transformer, the FEA can serve a vital role and may provide a cost effective and efficient solution to the problem. The preliminary results obtained from finite element model and the input impedance is calculated for the betterment of results the analysis must be completed by developing a three dimensional model only.

REFERENCES