Performance Analysis on Parallel Sparse Matrix Vector Multiplication Micro-Benchmark Using Dynamic Instrumentation Pintool

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Abstract

Sparse matrices have a variety of applications in today’s scientific and technological world, ranging from database systems, to image processing and machine learning. There are a lot of different algorithms and data structures that are used to implement these matrices in the computer, for efficient storage and computation. Sparse matrix vector multiplication (SPMV), in particular, is the most important operation performed using sparse matrices and is widely used in numerical analysis and solving equations. In this paper, we discuss about the major algorithms and formats that are used to store these sparse matrices. We have implemented these storage formats both sequentially and in parallel, and have observed that in both cases, the CSR format is the most efficient one amongst all. We also provide a performance analysis on sequential and parallel implementation of Sparse Matrix vector multiplication using the CSR format. Parallelization is done using OpenMP directive constructs. Cache analysis on sequential and parallel SPMV is performed using Pintool to depict how cache performance plays an important role in the performance improvement of a system.

Keywords: SpMV, OpenMP, Parallel, Pintool.

1. Introduction

Parallel computing is the recent form of computing, where many calculations are carried out simultaneously. The problem as such, is divided into many individual parts, and these parts are solved simultaneously. This improves time efficiency tremendously. This method is now prevalent in the form of multi-core processors. There are various hardware and software methods available to implement parallel computing. Hardware methods include using shared memory or using distributed (logically) memory. Software methods include the usage of various parallel programming languages such as OpenMP, MPI, CUDA, UPS, etc. Here, we used the OpenMP (Open Multiprocessing) software. When parallelizing, the speed difference between the processor and the memory presents some time lag. This constraint is also known as the memory wall problem. It can be rectified to a certain extent by extending the cache size, but it only works till the memory bandwidth is not the bottleneck in the performance.

Sparse matrices, matrices primarily populated with zeroes, are now in much use especially in the fields of combinatorics and network theory. Large sparse matrices are often used when solving partial differential equations. Since most of the values in a sparse matrix are zeroes, the space occupied by these is wastage of space. Various formats for storing only the non-zero values of these matrices were developed over the years, for more efficient storage and usage. The efficiency of these formats depends completely on the arrangement of the non-zero elements in the matrix. The various formats that are compared and discussed in this paper are Diagonal format, ELLPACK format, Coordinate format (COO), Compressed Sparse
Row (CSR) format, Blocked CSR format and Row-Group CSR format. Figure 1 gives the overview about this paper.

The rest of the paper is organised as follows: Section 2 explores various other works related to Sparse matrices. Section 3 gives a brief idea of the proposed system in the paper. Section 4 deals with the formats of sparse matrices that have been tested and a comparative study of these formats. Section 5 shows the concept of Sparse Matrix Vector Multiplication along with the algorithm used. Section 6 deals with PIN tools and OpenMP. Section 7 talks about the parallel performance parameters and cache performance parameters. Section 8 presents the results obtained and the observations made. The final section summarizes the main conclusions of this work.

![Flow Diagram](image)

Figure 1. Flow Diagram

## 2. Related Work

Sparse matrices, having a wide range of applications in various fields, have recently become an area rich in research. In the paper ‘Efficient Sparse Matrix-Vector Multiplication on CUDA’, Nathan Bell and Michael Garland [1] implement various data structures and algorithms for SpMV on CUDA platform, and emphasize on the memory bandwidth efficiency. H. Kotakemori et al in their paper ‘Performance Evaluation of Parallel Sparse Matrix-Vector Products on SGI Altix3700’ [2], focused on blocked formats of sparse matrices and how it is more advantageous to use these instead of the conventional formats (using OpenMP). Samuel Neves and Filipe Araujo [3] have proposed the idea of transforming the matrix into a straight line program, and hence making maximum usage of the instruction cache. Xiangzheng Sun et al[4] in their paper have proposed a new improved storage format for diagonal sparse matrices, called the Compressed Row Segment with Diagonal-Pattern(CSRD) and have implemented it on GPU using OpenCL. M.Krotiewski [5] et al performs massively parallel implementation of SpMV with scalar multi-core CPUs. Samuel Williams et al [6] in their paper have used SpMV, which is one of the algorithms where the kernels are heavily used, to test various computer architectures and have also given various optimization strategies. Dakuan et al [7], focused on the performance of SpMV, when performed with different number of processors. Here, we try to compare the performance of sequential and parallel SpMV over a range of sizes of matrices and used Pin tool to perform the cache analysis of both sequential and parallel SPMV.

## 3. Proposed System

Having taken the various formats of representing sparse matrices, we implemented the conversion of general sparse matrices of various sizes to these formats in C, and also found the time-efficiency for each, when implemented in sequential and in parallel. The time taken when executed in parallel had a significant decrease, hence giving more efficiency. We found that CSR by far had the best adaptability and efficiency for various kinds of matrices, as compared to all the above mentioned formats. Cache memory plays an important role in determining the efficiency of the system, as it reduces access time considerably. Pintool, which is used for dynamic program analysis, was used here to perform cache analysis using various cache performance parameters such as cache hit ratio, cache miss rate, etc. to determine the extent of improvement in efficiency when executed in parallel as compared to when executed sequentially.

## 4. Sparse Matrix Formats

The various formats of sparse matrices that are currently in use are (Table 1 gives a comparative study of the various formats discussed below):

### 4.1. Diagonal format

This format, though a very efficient one, in both time and space is not used quite often because it is not a general purpose matrix. It is applicable only if the non-zero elements are present as various diagonal elements, including the offsets.

It is implemented using two arrays- a 2-dimentional data array, that holds the non-zero elements, in which the various diagonals are now the columns, and another array for holding the offsets. [1]

For example:

\[
A = \begin{bmatrix}
0 & 4 & 0 & 7 & 0 \\
2 & 0 & 3 & 0 & 6 \\
0 & 5 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 \\
1 & 0 & 0 & 6 & 0
\end{bmatrix}
\]

\[
\text{Data} = \begin{bmatrix}
* & * & 4 & 7 \\
* & 2 & 3 & 6 \\
* & 5 & 0 & * \\
* & 0 & 2 & * \\
1 & 6 & 8 & *
\end{bmatrix}
\]

\[
\text{Offset} = [-4 -1 1 3 ]
\]

### 4.2. ELLPACK format

Another efficient, but not so general a format. This format is used for matrices obtained from semi-structured...
meshes, and well behaved unstructured meshes. It makes use of two 2-Dimentional matrices- one, for storing the non-zero elements (row-wise), and the other for storing the column index of the corresponding element in the first matrix. If the maximum number of non-zero elements in one row is \( m \), then, the size of these two arrays will be \( m \times m \). [8]

For example:

\[
A = \begin{pmatrix}
0 & 4 & 0 & 7 & 0 \\
2 & 0 & 3 & 0 & 6 \\
0 & 5 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 \\
1 & 0 & 0 & 6 & 0 \\
\end{pmatrix}
\]

Data = [4 7 * 2 3 6 5 * * 5 * * * * * * * * 1 6 * * 0 3 * *]

Indices = [1 3 * 0 2 4 1 4 0 3]

4.3. Coordinate (COO) format

This is one of the most general purpose matrices that can be implemented anywhere, without the fear of losing data. This of course, compromises on the efficiency with respect to space. It consists of three 1-Dimentional arrays, one to hold the non-zero elements, and the other two to hold the row-index and column-index of the corresponding element in the first array. We can have the row-array or the column-array in a sorted manner to enhance readability. (We have sorted the row-array in our code) [1]

For example:

\[
A = \begin{pmatrix}
0 & 4 & 0 & 7 & 0 \\
2 & 0 & 3 & 0 & 6 \\
0 & 5 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 \\
1 & 0 & 0 & 6 & 0 \\
\end{pmatrix}
\]

Data = [4 7 2 3 6 5 2 1 6]

Row = [0 0 1 1 1 2 3 4 4]

Column = [1 3 0 2 4 1 4 0 3]

4.4. Compressed Row (CSR) format

This is the most popularly used format, as it is both a general purpose format, and is very efficient. There have been a lot of other improved versions of this format, like the blocked CSR and the row-grouped CRS (described later), that have been introduced and implemented. It is implemented using three 1-Dimentional arrays- a pointer row, that holds the number of non-zero elements in each row, the second one to hold the non-zero elements, and the last one to hold the column indices of the corresponding element in the second matrix. The sizes of the matrices are purely dependent on the number of non-zero elements in the matrix, and hence no matter how many zeroes are padded, the size occupied will remain the same. The row pointer array facilitates fast multiplication, and the code remains unchanged for SpMV [1].

For example:

\[
A = \begin{pmatrix}
0 & 4 & 0 & 7 & 0 \\
2 & 0 & 3 & 0 & 6 \\
0 & 5 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 \\
1 & 0 & 0 & 6 & 0 \\
\end{pmatrix}
\]

Data = [4 7 2 3 6 5 2 1 6]

Indices = [1 3 0 2 4 1 4 0 3]

Ptr = [0 2 5 6 7 9]

4.5. Row-grouped CSR format

This is an improvised version of the CSR format. The authors decompose the given sparse matrix into two halves, row-wise. The first occurrences of non-zero data in each row, in the first half are put into a 1-D array, followed by the next occurrences. This is then repeated for the second half of the matrix also. If the number of non-zero data in any row is less than the maximum number of non-zero elements in each row in the matrix, the array is padded with zeroes [8].

For example:

\[
A = \begin{pmatrix}
4 & 5 & 7 & 3 & 0 & 0 & 6 & 0 & 2 & 1 & 4 & 0 & 6 & 0 \\
2 & 0 & 3 & 0 & 6 \\
0 & 5 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 \\
1 & 0 & 0 & 6 & 0 \\
0 & 0 & 4 & 0 & 0 \\
\end{pmatrix}
\]

Values = [4 2 5 7 3 0 0 6 0 2 1 4 0 6 0]

Columns = [1 0 1 3 2 * * 4 * 4 0 2 * 3]

Row-Lengths = [2 3 1 1 2 1 1]

Group Pointers = [0 9]

4.6. Blocked Sparse Row (BSR) format

This is another improvised version of the CSR format. The authors divide the given sparse matrix into blocks, depending on the concentration of the non-zero values. It makes use of 4 one-dimensional arrays. One to store the non-zero values, one to store the column number of the first non-zero value in each block, one to store the position of the point where each block in a row starts, and the last one to store the location of the first non-zero element in each block [1].
Table 1 - Comparison Of Various Formats Of Sparse Matrices

<table>
<thead>
<tr>
<th>Categories</th>
<th>Number of matrices required</th>
<th>Usage</th>
<th>Advantage</th>
<th>Disadvantage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagonal Matrix</td>
<td>2 (1 2D and 1 1D array)</td>
<td>Not a general purpose format</td>
<td>For a matrix with non-zero elements only in the diagonals, it is very effective</td>
<td>It can be applicable only for matrices that have non-zero elements only in its diagonals</td>
</tr>
<tr>
<td>ELLPACK format</td>
<td>2 2D arrays</td>
<td>Less general purpose format</td>
<td>Suitable for matrices obtained from semi-structured meshes, and well behaved unstructured meshes</td>
<td>We need to know the maximum number of non-zero elements present in the matrix.</td>
</tr>
<tr>
<td>Coordinate form</td>
<td>3 1D arrays</td>
<td>General purpose format</td>
<td>It is suitable for any random sparse matrix</td>
<td></td>
</tr>
<tr>
<td>CSR</td>
<td>3 1D arrays</td>
<td>General purpose format</td>
<td>They reduce storage. Row pointers facilitate fast multiplication. The code need not be changed for SpMV implementation.</td>
<td></td>
</tr>
<tr>
<td>Row-grouped CSR</td>
<td>4 1D arrays</td>
<td>General purpose format</td>
<td>Allocates less artificial zeros, number of allocated elements per row may vary from one group to an other</td>
<td>It is a very time consuming process and it requires 4 arrays.</td>
</tr>
<tr>
<td>Blocked compressed row storage</td>
<td>4 1D arrays</td>
<td>General purpose format</td>
<td>Reduces the number of load operations</td>
<td>It requires an extra loop in the sparse matrix vector multiplication and suffers from additional loop overhead</td>
</tr>
</tbody>
</table>

For example:

\[
A = \begin{pmatrix}
5 & 7 & 0 & 0 & 0 \\
0 & 2 & 1 & 0 & 0 \\
0 & 1 & 0 & 2 & 0 \\
0 & 0 & 1 & 4 & 0 \\
3 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

Data = [5, 7, 2, 1, 1, 2, 1, 4, 3]
Col = [1, 2, 2, 4, 3, 1]
Row = [1, 2, 3, 5, 6]
NZ = [1, 3, 5, 6, 8, 9]

5. Sparse Matrix Vector Multiplication

Sparse matrix vector multiplication [2][7] is just multiplying a sparse matrix with a single dimensional matrix, to get the resultant single dimensional array. This is the most important and widely used operation done using sparse matrices. Solving partial difference equations and eigen value problems require thousands of such multiplication operation to reach convergence. This requires a lot of computation that will in turn require a considerable amount of time. Hence, parallelization of these SpMV algorithms are done to decrease the execution time of calculation and hence make the process more efficient.

For Example:

\[
\begin{pmatrix}
\alpha_{11} & \alpha_{12} & \ldots \\
\alpha_{21} & \alpha_{22} & \ldots \\
\ldots & \ldots & \ldots
\end{pmatrix}
\times
\begin{pmatrix}
b_{11} \\
b_{12} \\
\ldots
\end{pmatrix}
= 
\begin{pmatrix}
c_{11} \\
c_{12} \\
\ldots
\end{pmatrix}
\]

There are however, a few problems [8] associated with sparse matrix vector multiplication that affects its performance. Firstly, there is no temporal locality in sparse matrices, making it difficult to place the values necessary in the cache. Moreover, each value is used just once. Secondly, as various formats are used to store only the non-zero values to save memory, a lot of indirect memory references are made during execution that leads to additional load operations and cache interference. Finally, the memory access of the sparse matrices is very random and irregular and depends on the sparsity of the structure.

The code for implementing sparse matrix vector multiplication using the Compressed Row (CSR) format is:

**Code:**

MULTIPLY( int size)

x: input array
y: Result
ptr: Pointer array in the CSR format
col: Column array in the CSR format
for i from 0 to (size-1)
    y[i]=0
    for j from ptr[i] to ptr[i+1]
This was implemented for matrices of various sizes up to 1500*1500, both sequentially and in parallel.

6. OpenMP

OpenMP (Open Multiprocessing) software is an API (Application Programming Interface) that supports multi-platform shared memory multiprocessing programming in C, C++, and FORTRAN, on most processor architectures and operating systems. It works with fork-join method. The compiler, on receiving the code, generates the multi-threaded version of this code, using the directives. Each thread is given to a separate core, where they are executed simultaneously. In the end, the main thread gives the result. It is preferred here over the other available software because of various reasons; firstly, it has portable multithreaded code, and has unified code for both sequential and parallel implementation (the OpenMP constructs are treated as comments when run sequentially). Secondly, it is very simple to use as it does not deal with message passing, unlike MPI. It is also very easily understandable. Finally, the compiler directives that are used for achieving parallelism can be easily embedded in C/C++ source code.

OpenMP accomplishes parallelism exclusively by means of threads. It is an explicit programming model and allows the user full control over parallelization. Here, we use OpenMP in Linux environment, which requires the gcc or the g++ compiler. The flag used here is, -fopenmp. To parallelize a program using OpenMP, first, the code is divided into two parts- the part that can be parallelized, and the part that cannot. The part that can be parallelized is then examined and dependent variables are identified.

6.1 Pintool

Pin tools are used to perform program analysis, for analysing dynamic behaviour [9][10]. Figure 2 gives the block diagram of the working of the PINtool. Initially used for computer architecture analysis, it has been extended, and now can be used as an API for security and parallel program analysis. Various hardware parameters such as number of memory accesses, cache hits and cache misses can be derived using these pin tools, which can be used to make comparisons to decide the most efficient algorithm and can also be used for testing the efficiency of the underlying architecture.[7]

7. Performance Parameters for Parallel Implementation

When implementing programs sequentially, i.e., on a single core, the performance can be enhanced using techniques like flexible and sequential random access memory, using prefetch instructions to improve the cache performance [11]. In the ‘Software Prefetch on Core Micro-Architecture Applied to Irregular Codes’ paper, the authors have used software prefetch instructions in combination with hardware prefetch unit, which showed a maximum improvement of 40% in execution time.

The reason for opting for parallel computing is to make a program more efficient and better utilization of the current hardware resources. Hence, analysis is very important to check if the parallel implementation [12] is indeed better than the previous implementation. The first and obvious parameter is the time taken to execute the program, or the Execution time. By directly checking if the time taken to solve the problem in parallel execution takes less time, we can measure its effectiveness. The other parameters used to check the performance are:

7.1 Speedup

It is the ratio of the time taken to implement the program sequentially, to the time taken to implement the program using parallel computing.

It is given as:

\[ S = T_s / T_p \]

Ideally, the speedup is equal to the number of threads used (p).

But theoretically, the speedup can never be greater than the number of threads used. In some practical cases, the speedup is more than p.

7.2 Efficiency

It is defined as the ratio of the speed up, to the number of threads used (p).
E = S / p
Ideally, the efficiency is equal to one.
But practically, it lies in the range of zero to one.

7.3. Scalability

It is the ability of the machine to incrementally expand, and to allow the incorporation of more processors in the interconnecting network, without degrading the communication speed.

In this paper, the speed up and the efficiency were calculated for SpMV implementation for various sizes of matrices.

8. Cache performance Parameters

The efficiency of the various methods of sparse matrix vector multiplication can be tested using the cache performance parameters. The placement and replacement of data in the cache depends on various parameters and various algorithms used. The time taken by the processor to access data is an important parameter of the total time consumed by the process. Hence, the required data needs to be present in the cache to minimize the time consumption. The various cache performance parameters are:

8.1. Cache hit ratio (h)

It is the ratio of the number of cache hits to the total number of references to the cache.
Typically, it lies between 0.90 and 0.97.

8.2. Cache miss rate

It is the ratio of the number of cache misses to the total number of references made to the cache.
It is simply equal to: 1 – HitRatio.

8.3. Average Memory Access Time

It is defined as: h * T_{cache} + (1-h) T_{mem}.
Where, T_{cache} is the time taken to access the cache, and T_{mem} is the time taken to access the memory.
These parameters were calculated using the cache statistics that were obtained by using pintool, and were used to compare the results that were obtained when the program was executed sequentially and in parallel.

9. Results and Observations

The processor used for implementation purposes is Intel processor, Nehalem. It has a L3 cache of 4–12 MB, a 256 KB L2 cache and a 64KB L1 cache core (32 KB L1 data and 32 KB L1 instruction). It has an integrated memory controller that supports two to three memory channels of DDR3 SDRAM. [13] All the programs were implemented in C language in Linux platform. The Linux operating system used was Ubuntu, version 11.10. Figure 3 shows the block diagram of the Nehalem core.

![Figure 3. Block Diagram of Nehalem](image)

<table>
<thead>
<tr>
<th>Size</th>
<th>ELLPACK</th>
<th>COO</th>
<th>CSR</th>
</tr>
</thead>
<tbody>
<tr>
<td>8*8</td>
<td>0.31</td>
<td>0.31</td>
<td>0.15</td>
</tr>
<tr>
<td>10*10</td>
<td>0.47</td>
<td>0.16</td>
<td>0.15</td>
</tr>
<tr>
<td>12*12</td>
<td>0.63</td>
<td>0.15</td>
<td>0.16</td>
</tr>
<tr>
<td>14*14</td>
<td>0.78</td>
<td>0.31</td>
<td>0.31</td>
</tr>
<tr>
<td>16*16</td>
<td>0.74</td>
<td>0.42</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Various sparse matrix storage formats were implemented in sequential and parallel. Sample results are given in Table 2. The corresponding graph is given in Figure 4.

![Figure 4. Graph of execution times of various formats](image)

Having observed that CSR format is the most efficient amongst the considered formats, we have implemented SpMV using CSR format for various sizes in the given architecture, both serially and in parallel.

9.1 Execution times obtained for SpMV using CSR format

Table 3 shows the results obtained when SpMV is implemented in parallel and sequentially. As the size of the matrix increases, the execution time also increases accordingly. It is apparent from the readings that the time taken for execution has decreased considerably when implemented in parallel, as compared to sequential. Figure 5, which is the graphical representation of the result, clearly demonstrates this difference in execution times. The bigger the size of the matrix, the greater the difference. Next, we used the pin tool to calculate the memory access statistics.
Table 3 - Comparison of Execution Times

<table>
<thead>
<tr>
<th>Matrix Size</th>
<th>Sequential Timing</th>
<th>Parallel Timing</th>
</tr>
</thead>
<tbody>
<tr>
<td>100*100</td>
<td>58</td>
<td>0.468</td>
</tr>
<tr>
<td>200*200</td>
<td>64</td>
<td>0.650</td>
</tr>
<tr>
<td>500*500</td>
<td>91</td>
<td>1.395</td>
</tr>
<tr>
<td>700*700</td>
<td>2810</td>
<td>3.568</td>
</tr>
<tr>
<td>1000*1000</td>
<td>2989</td>
<td>4.054</td>
</tr>
<tr>
<td>1500*1500</td>
<td>3216</td>
<td>4.558</td>
</tr>
</tbody>
</table>

Table 4 - Data Cache Statistics

<table>
<thead>
<tr>
<th>Mode of Execution</th>
<th>Total Accesses</th>
<th>Total Hits</th>
<th>Total Misses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential</td>
<td>17812399</td>
<td>15676617</td>
<td>2135782</td>
</tr>
<tr>
<td>Parallel</td>
<td>20838197</td>
<td>18666759</td>
<td>2171438</td>
</tr>
</tbody>
</table>

Table 5 - Performance Parameters

<table>
<thead>
<tr>
<th>Mode of Execution</th>
<th>Hit Ratio</th>
<th>Miss Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential</td>
<td>0.880095</td>
<td>0.119904</td>
</tr>
<tr>
<td>Parallel</td>
<td>0.8957996</td>
<td>0.104205</td>
</tr>
</tbody>
</table>

Table 6 - Comparison Of Cache Performance Parameters

<table>
<thead>
<tr>
<th>Mode of Execution</th>
<th>Cache Hit</th>
<th>Miss Rate</th>
<th>Execution Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential</td>
<td>Low</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Parallel</td>
<td>High</td>
<td>Low</td>
<td>Low</td>
</tr>
</tbody>
</table>

for data cache for sequential and parallel implementation. The total memory accesses, the total cache hits and misses are noted. Table 4 and Figure 6 give the observations made when 1000*1000 matrix SpMV was implemented both sequentially and in parallel.

With the made observations, we calculated the hit ratio and the miss rate for data cache. Table 5 gives the values calculated and Figure 7 is the graphical representation of these values, that gives the comparison between sequential and parallel implementation based on these parameters.

It is clear, that the number of hits and the hit ratio is higher when the program is implemented in parallel, than when done sequentially. Table 6 gives the overall inference from the results obtained.
10. Conclusion

We have, in this paper, demonstrated the implementation of SpMV both sequentially and in parallel using OpenMP. Having compared the various storage formats of sparse matrices, we have observed that the CSR format is the most efficient general purpose storage format. Having observed so, we then implemented sparse matrix vector multiplication using this format both sequentially and in parallel and found that parallelizing SpMV leads to a considerable improvement in its efficiency, in terms of both execution time and cache usage.

References