A Novel Approach for Extraction of Blood Vessels from Medical Images using Fuzzy Mathematical Morphology and Adaptive Sequential Filtering

Sreekala. K*, Sreelija V*

* Christ University, Bangalore, India.
kannothsree@gmail.com
* Wipro Technologies, Bangalore, India.
sreelija.velluva@wipro.com

Abstract- This paper presents a method for extraction of blood vessels from medical images using Fuzzy Mathematical Morphological operations along with adaptive sequential filtering. The extraction of blood vessels from medical images plays an important role in medical analysis and diagnosis. Here the image and structuring elements are considered as fuzzy sets, so the fast implementation of Mathematical Morphology is possible. The basic idea is to replace the fixed size structuring elements by variable size structuring elements. First the segmentation is done using fuzzy morphological techniques, then a sequence of adaptive sequential closing and opening are performed in order to eliminate the background noise. The approach has been tested on a series of retinal images and results shows that it is a very promising and effective approach.

I. INTRODUCTION

Angiography is an important diagnostic test in which detailed photographs of the retina and choroid are taken after injecting a dye into an arm vein. Role of retinal angiographic image is important in detection and diagnosis of eye diseases like Glaucoma, diabetic retinopathy, Macular degeneration etc. These diseases are very serious and they may lead to blindness if they are not detected and treated properly[1]. Information about blood vessels help to detect and diagnose the disease and also it gives information about the intensity of the disease. But the manual detection of blood vessel is a difficult task because of the complexity of angiographic images. So it is necessary to have an automatic method for the extraction of blood vessels from the angiographic images.

Mathematical Morphology, the foundation of Morphological image processing, is based on set theory. Initially it was developed for binary images, and later got extended to gray scale images. It relies on a set of pixel locations called structuring elements which is convolved over the image. It is a special mask filter which is used to enhance the input image. In Fuzzy Mathematical Morphology the image and structuring elements are gray scale. Gray scale images can be represented as fuzzy sets. So these fuzzy concepts can be used in Traditional Mathematical Morphology which leads to Fuzzy Mathematical Morphology. This work uses Fuzzy Mathematical Morphology as the basic root to extract the blood vessels.

Another important point that has to be considered here is the structuring element. Unlike the Traditional Mathematical Morphology where the structuring elements are taken as images, the Fuzzy Mathematical Morphology takes structuring element as fuzzy sets which is obtained from the image itself. In the proposed method the image and structuring element are converted to fuzzy sets and the basic morphological operations erosion, dilation, and opening are performed to extract the vascular tree. Then a sequence of sequential filtering operations like opening and closing are performed to remove the unwanted portions.

II. PROPOSED WORK

A. Fuzzy Mathematical Morphology

If two predicates[2] P and Q are given, then the basic operators used in fuzzy set theory are

Conjunction \( P \land Q \),

Disjunction \( P \lor Q \),

Negation \( \neg P \) and

Implication \( \neg P \lor Q \).

The most Commonly used Conjunction and Implication formulae are given below.

Godel Brouwer:

\[
C(a,t) = a \land t
\]

\[
I(a,s) = \begin{cases} 
 s, & s < a \\
 1, & s \geq a 
\end{cases}
\]

Kleene Dienes:

\[
C(a,t) = \begin{cases} 
 0, & t \leq 1 - a \\
 t, & t > 1 - a 
\end{cases}
\]

\[
I(a,s) = 1 - a \lor s
\]

In a gray scale image, Each gray level is associated with a value between 0 and 1. Fuzzy Morphological operators can be defined by means of fuzzy logic[3].
Let A and B belong to the set of image parts, then

\[ A \subseteq B \iff \forall y \in f, \quad y \in A \Rightarrow y \in B \]
\[ \iff \forall y \in f, \quad A(y) \Rightarrow B(y) \]
\[ \iff \forall y \in f, \quad I(A(y), B(y)) = 1 \]

Where I denote the binary Implication.

The fuzzy erosion of an image \( f \) by a structuring element B at a point \( x \) is given by

\[ e^f_x(f, B)(x) = \inf_{y \in f} \{ I(B(y)), f(y) \} \tag{1} \]

Similarly, being A and B part of image \( f \), then

\[ A \cap B \neq \phi \iff \exists y \in f \quad y \in A, \quad y \in B \]
\[ \iff \exists y \in f, C(A(y), B(y)) = 1 \]

where \( C \) is binary conjunction.

Then the fuzzy dilation of an image \( f \) by a structuring element at a point \( x \) is defined as

\[ \delta^f_x(f, B)(x) = \sup_{y \in f} \{ C(B(y)), f(y) \} \]

From morphological theory, fuzzy opening is described as

\[ y^f(f, B) = \delta^f(e^f(f, B), B) \tag{3} \]

and fuzzy closing as

\[ \Phi^f(f, B) = e^f(\delta^f(f, B), B) \tag{4} \]

Therefore the fuzzy inner edge is defined as

\[ \delta^f_I(f, B) = f - e^f(f, B) \tag{5} \]

and the fuzzy outer edge is defined as

\[ \delta^f_O(f, B) = \delta^f(f, B) - f \tag{6} \]

Fuzzy Top-Hat by opening is

\[ Top - Hat^f(y) = f - y^f(f, B) \tag{7} \]

and Fuzzy Top-Hat by closing is

\[ Top - Hat^f(y) = \Phi^f(f, B) - f \tag{8} \]

### B. Adaptive Morphological Operators and Filters

The elementary dual operators of adaptive dilation and erosion are defined as[4]

\[ \forall (m, h, f) \in \mathbb{R}^+ \times \mathbb{C} \times I \]

\[ \delta^h_m(f) : \begin{cases} \delta \rightarrow \mathbb{R} \\ x \mapsto \sup_{w \in \mathbb{R}^+} f(w) \tag{9} \end{cases} \]

\[ \epsilon^h_m(f) : \begin{cases} \delta \rightarrow \mathbb{R} \\ x \mapsto \inf_{w \in \mathbb{R}^+} f(w) \tag{10} \end{cases} \]

From the lattice theory[5] adaptive morphological filtering operations i.e; closing and opening are respectively defined as

\[ \forall (m, h, f) \in \mathbb{R}^+ \times \mathbb{C} \times I \]

\[ \Phi^h_m(f) : \begin{cases} \delta \rightarrow \mathbb{R} \\ x \mapsto \epsilon^h_m \circ \delta^h_m(f)(x) \tag{11} \end{cases} \]

\[ \gamma^h_m(f) : \begin{cases} \delta \rightarrow \mathbb{R} \\ x \mapsto \delta^h_m \circ \epsilon^h_m(f)(x) \tag{12} \end{cases} \]

### C. Adaptive Sequential Morphological Filters

The adaptive morphological filters described by equations (11) and (12) are generally neither size distribution nor anti-size distribution. Sequential filters are built by naturally reiterate adaptive dilation or erosion. So the adaptive sequential dilation, erosion, closing and opening are respectively defined as[6]

\[ \forall (m, p, h) \in \mathbb{R}^+ \times N \times C \]

\[ \delta^h_{mp} : \begin{cases} I \rightarrow I \\ f \mapsto \delta^h_m \circ \cdots \circ \delta^h_1(f) \tag{13} \end{cases} \]

\[ \epsilon^h_{mp} : \begin{cases} I \rightarrow I \\ f \mapsto \epsilon^h_m \circ \cdots \circ \epsilon^h_1(f) \tag{14} \end{cases} \]

\[ \Phi^h_{mp} : \begin{cases} I \rightarrow I \\ f \mapsto \epsilon^h_m \circ \cdots \circ \epsilon^h_1(f) \tag{15} \end{cases} \]

\[ \gamma^h_{mp} : \begin{cases} I \rightarrow I \\ f \mapsto \delta^h_m \circ \epsilon^h_m(f) \tag{16} \end{cases} \]

The morphological duality of \( \delta^h_{mp} \) and \( \epsilon^h_{mp} \) provides the two sequential morphological filters \( \Phi^h_{mp} \) and \( \gamma^h_{mp} \). Thus the extension of well known alternating sequential filters can be defined as

\[ ASFOC^h_{mp}(f) : \begin{cases} I \rightarrow I \\ f \mapsto \gamma^h_{mp} \circ \cdots \circ \gamma^h_{mp}(f)(x) \tag{17} \end{cases} \]

\[ ASFCO^h_{mp}(f) : \begin{cases} I \rightarrow I \\ f \mapsto \Phi^h_{mp} \circ \cdots \circ \Phi^h_{mp}(f)(x) \tag{18} \end{cases} \]

### III. PROPOSED ALGORITHM

The steps followed in the proposed method are

**Step1:** In the first step the gray scale image pixels are converted to fuzzy values i.e; fuzzification of the image[7]. It is done by using the function

\[ \theta(t) = \frac{1}{2} + \frac{1}{\pi} \arctan(t) \tag{19} \]
Step 2: Fuzzy opening operation is performed using equation (3), by means of Kleene-Dienes conjunction. The structuring element used here is a 3-D cone shape structuring element [2]. Its values range from 0 to 1, and it is obtained from the following formula:

\[
\frac{1}{f(x,y)} \begin{vmatrix}
    f(x,y-2j) & f(x,y-j) & f(x,y)-2i & f(x,y)-i & f(x,y)-2j \\
    f(x,y-j) & f(x,y) & f(x,y)-i & f(x,y)-i & f(x,y)-2j \\
    f(x,y)-2i & f(x,y)-i & f(x,y) & f(x,y)-i & f(x,y)-2j \\
    f(x,y-2i) & f(x,y-i) & f(x,y) & f(x,y)-i & f(x,y)-2j \\
    f(x,y-j) & f(x,y)-i & f(x,y)-2i & f(x,y)-i & f(x,y)-2j \\
\end{vmatrix}
\]

If any value is below zero, it is assigned a zero value.

Step 3: In the third step calculation of fuzzy Top-Hat transform is done. It is obtained by subtracting the fuzzy opened image from the original image. This extracts the locally brilliant elements from the image.

Step 4: Fourth step is the image purification process, where Adaptive sequential filtering operation is performed by an alternating sequential filter with Opening and Closing (ASFOC) which is given by the equation (17). Here the Top-Hat transformed image is filtered sequentially for more than three times to eliminate the noise.

Step 5: The resultant image is defuzzified using the inverse function of equation (19). Defuzzification process takes place in order to convert the fuzzy partition matrix to a crisp partition so that a binary image with only the vascular tree can be visualised.

IV. RESULTS AND DISCUSSION

The proposed method was applied to many retinal angiographic images. This type of images accounts for a great deal of noise which leads to notorious difficulty when it comes to applying other traditional segmentation techniques. When processing these images with the proposed algorithm, the lines of interest were well segmented. Few results are shown below (figure (1)) and the efficiency of the proposed method can be clearly appreciated (figure (2)).

V. CONCLUSIONS

Here, method for the extraction of retinal image vessels is presented. The method is developed by using mathematical morphology combined with the fuzzy techniques. These techniques are first employed to smooth and strengthen the retinal images as well as to suppress the background information. Then, it enhances retinal images. Finally, a purification procedure is introduced using adaptive sequential filters and defuzzification is done. The blood vessels are then extracted. The detection results obtained from the proposed method are compared with other segmentation methods.

Fig. 1: (a) Original image (b) fuzzified image (c) eroded image (d) opened image (e) tophat by opening (After sequential filtering) (f) defuzzified image

Fig. (2). Comparison: (a) Using fuzzy techniques with Adaptive sequential filters (b) Using only Top-Hat transform
REFERENCES


