An Integrated Support Vector Machine and Quantum Behaved Particle Swarm Optimization Algorithm for Groundwater Level Forecasting

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Abstract—Groundwater level prediction in a water basin plays a significant role in the management of groundwater resources. A groundwater level forecasting system is developed in this study using Support vector Machines (SVM). Further Quantum behaved Particle Swarm Optimization (QPSO) function is employed in this study to determine the SVM parameters. Later, the proposed SVM-QPSO model is used in determining the groundwater level of Visakhapatnam region of Andhra Pradesh in India. The performance of the SVM-QPSO model is then compared with the ANN (Artificial Neural Networks). The results indicate that SVM-QPSO is a far better technique for predicting groundwater levels as it provides a high degree of accuracy and reliability.

Keywords: SVM, groundwater level Forecasting, Time series, QPSO

I. INTRODUCTION

An accurate prediction of the hydrological processes such as precipitation and groundwater level plays a significant role in providing information for planning, land use and water resources management (1). These hydrological processes are influenced by many factors such as weather, soil moisture, infiltration and evapotranspiration to name a few. These processes are thus very complex in nature and are highly nonlinear in their characteristics. Modeling these processes accurately has been a challenge since the past few decades. In this study, an attempt has also been made to develop a forecasting model which could predict groundwater levels. In the recent decades, machine learning models are employed in modeling non-linear processes that are complex in nature (2), particularly in problems of flow forecasting where artificial neural network has been widely used (3). Besides neural networks, other very recent techniques such as support vector machines (4) and genetic programming (5) have been found to be effective in modeling virtually any nonlinear function. SVM application studies are done widely by ‘expert’ users (6). The performance of SVM models greatly depends on SVM parameter selection, so lot of research is going on to find out the best parameters for a given data set in SVM regression. Till now there is hardly any consensus in choosing the SVM parameters. Further there are many contradictory opinions regarding the selection of the parameters. Thus, re-sampling remains to be the possible method for selection of parameters. Unfortunately, using re-sampling proves to be very expensive in terms of computational costs and data requirements for tuning several SVM parameters. Thus, the main aim of this study is thus to find out the ways and means for obtaining the suitable parameters for the SVM model. In this study Quantum behaved Particle swarm optimization (PSO) technique is employed to find out SVM parameters.

Eberhart and Kennedy (7) was the first to introduce PSO. Since then PSO has undergone many improvements (8). QPSO (Quantum-behaved PSO) is also one such attempt done by Sun and Xu (9) to improve the performance of the PSO by introducing the principles of quantum theory in finding the good optimal solutions. QPSO has been widely tested on some standard benchmark functions and it shows that QPSO outer performs compared to standard PSO (9) thus proving to be a promising algorithm. In this study QPSO technique is adapted to determine SVM parameters.

II. SUPPORT VECTOR MACHINES

Support vector machine (SVM) is a new and
promising technique for data classification and regression. In this section a brief description of SVM is given. Assume \(\{(x_i, y_i), \ldots, (x_i, y_i)\}\) be the given training data sets, where each \(x_i \in \mathbb{R}^n\) shows the input space of the sample and has a corresponding target value \(y_i \in \mathbb{R}\) for \(i = 1, \ldots, \ell\). Where \(\ell\) represents the size of the training data. The support vector regression solves an optimization problem:

\[
\text{Minimize} \quad \frac{1}{2} \|\omega\|^2 + C \sum_{i=0}^\ell (\xi_i + \xi_i^*) \quad (1)
\]

Subjected to

\[
\begin{align*}
\left\{ \begin{array}{l}
y_i - (\omega, x_i) + b - \xi_i^* \\
\xi_i^* \geq 0, i = 1, \ldots, \ell
\end{array} \right.
\end{align*}
\]

\[
\sum_{i=0}^\ell (\xi_i + \xi_i^*) \quad (2)
\]

where \(x_i\) is mapped to a higher dimensional space by the function \(\varphi\), \(\xi_i\) is the upper training error (\(\xi_i^*\) is the lower) subject to the insensitive tube \(y_i - (\omega, x_i) + b = \xi_i^*\). The parameters which control the regression quality are the cost of error \(C\), the width of the tube, and the mapping function \(\varphi\). The constraints imply that we would like to put most data \(x_i\) in the tube \(y_i - (\omega, x_i) + b \leq \xi_i\). If \(x_i\) is not in the tube, there is an error \(\xi_i\) or \(\xi_i^*\) which we would like to minimize in the objective function. SVM avoids under fitting and over fitting the training data by minimizing the training error \(C \sum_{i=0}^\ell (\xi_i + \xi_i^*)\) as well as the regularization term \(\|\omega\|^2\). For traditional least square regression is always zero and data are not mapped into higher dimensional spaces. Hence SVM is a more general and flexible treatment on regression problems.

Many works in forecasting have demonstrated the favourable performance of the radial basis function (6; 10; 11) as Kernel function for SVM. Therefore, The Radial Basis Function (RBF), \(\exp(-\gamma \|x - x_i\|^2)\) is adopted in this work. The selection of the three parameters \(\gamma, \sigma\), and \(C\) of SVM model influence the accuracy of forecasting. However there is no standard method of selection of these parameters. Therefore, Quantum Behaved Particle Swarm Optimization technique is used in the proposed model to optimize parameter selection.

III. QUANTUM BEHAVED PARTICLE SWARM OPTIMIZATION TECHNIQUE IN SELECTING THE PARAMETERS OF SVM

PSO has been a popular method for the problem of global optimization of functions in continuous space primarily because of its simple implementation and less computational demands. However, its convergence to the global optimum is not guaranteed. Therefore a large number of improvements upon the original PSO have been proposed since its development. A Quantum-behaved Particle Swarm Optimization (QPSO), inspired from quantum mechanics and based on delta-potential-well model, was proposed by Sun, Xu and Feng (9). The salient features of QPSO, compared to PSO, include a reduction in the number of parameters and superior performance on a variety of functions in multi-dimensional continuous space.

In the quantum physics, the state of a particle with momentum and energy can be depicted by its wave function \(\psi(x, t)\). According to QPSO theory each particles in a quantum state and is formulated by its wave function \(\psi(x, t)\) instead of the position and velocity which are in PSO. According to the statistical significance of the wave function, the probability of a particle’s appearing in a certain position can be obtained from the probability density function \(|\psi(x, t)|^2\). And then the probability distribution function of the particle’s position can be calculated through the probability density function. By employing the Monte Carlo method, the particle’s position is updated according to the following equation:

\[
x_i^{t+1} = p_i^t \pm 0.5 \cdot L_i^t \cdot \ln \left( \frac{1}{u_i^t} \right)
\]

where \(u_i^t\) is a random number uniformly distributed in \((0, 1)\); \(p_i^t\) is the and defined as local attractor and defined as \(p_i^t = \varphi p_i^t + (1 - \varphi p_i^t) \cdot p_i^t\) where \(\varphi^t\) is a random number uniformly distributed in \((0, 1)\). In parameter \(L_i^t\) is evaluated by

\[
L_i^t = 2 \cdot \beta |p_i^t - x_i^t|
\]

where parameter \(\beta\) is called the contraction-expansion (CE) coefficient, which can be tuned to control the convergence speed of the algorithms. Then we get the position update equation as

\[
x_i^{t+1} = p_i^t \pm \beta |p_i^t - x_i^t| \cdot \ln \left( \frac{1}{u_i^t} \right)
\]

The PSO algorithm with position update equation (5) is called as quantum delta-potential-well-based PSO (QDPSO) algorithm. Keeping in view the vital position of \(L\) for convergence rate and performance of the algorithm an improvement was proposed to evaluate parameter \(L\). As per this algorithm the mean best position \((\text{mbest})\) is defined as the center of best positions of the swarm. That is

\[
\text{mbest} = \left( \frac{1}{M} \sum_{i=1}^{M} p_i^1, \frac{1}{M} \sum_{i=1}^{M} p_i^2, \ldots, \frac{1}{M} \sum_{i=1}^{M} p_i^L \right)
\]

where \(M\) is the population size and \(P_i^t\) is the personal
best position of particle. Further Parameter L is given by 
\[ L_{ij}^t = 2 \cdot \beta \cdot mbest_j^t - X_{ij}^t \]  
(7)
Hence, the particle’s position is updated according to the following equation:
\[ X_{ij}^{t+1} = p_{ij}^t + \beta \cdot \left( mbest_j^t - X_{ij}^t \right) \cdot \ln \left( \frac{1}{u_{ij}} \right) \]  
(8)
The PSO algorithm with equation (8) is called as quantum-behaved particle swarm optimization (QPSO). Pseudo code for implementing the QPSO is given below:
Initialize the population size, the positions, and the dimensions of the particles
for \( t = 1 \rightarrow \) Maximum Iteration \( T \) do
Compute the mean best position \( mbest \)
\[ \beta = (1.0 - 0.5) \ldots (T - t)/T + 0.5 \]
for \( i = 1 \rightarrow \) population size \( M \)
if \( f(x_i) < f(p_i) \) then
\[ P_i = X_i \]
end if
\[ P_i = \min(p_i) \]
for \( j = 1 \rightarrow \) dimension \( D \) do
\[ \Delta = \text{rand}(0, 1); u = \text{rand}(0, 1); \]
\[ p_i = \Delta \cdot P_i + (1 - \Delta) \cdot P_u \]
if rand(0, 1) > 0.5 then
\[ X_i = p_i + \beta \]
else
\[ X_i = p_i - \beta \cdot (mbest - X_i) \cdot \log(1/u) \]
end if
end for
end for
IV. APPLICATIONS AND DISCUSSIONS
A. Description of the Study area
Visakhapatnam city is located in Andhra Pradesh along the East Coast of India at latitude \( 17^\circ45' \) North and longitude \( 83^\circ20' \) East. The city is known for its rapid industrial expansion and therefore subjected to land use changes simultaneously. While diversion of surface water in huge quantities is aimed as a solution to meet the industrial needs, the domestic is planned to be met from groundwater reservoirs. The rapid growth in the number of apartments that are being constructed in the city and the subsequent demand for groundwater, which is the main resource for the inhabitants of these, are causing gradual decline of availability of water. Also there are indications of salinity intrusion in the areas proximate to sea. This situation has thus become a problem of concern to be solved. The Asia’s second biggest residential colony Muvvala vanni palem known as MVP colony till recently had stable groundwater table, but now not so due to increasing population density. Hence this area has been selected for the present study.

Fig. 1 Wells under study shown on DEM of the study Area

B. Model Development
The root mean square error (RMSE), Mean Absolute Percentage Error (MAPE) Nash-Sutcliffe efficiency coefficient (EFF) and correlation coefficient (CORR) are used for evaluating the performance of the developed models. In this study, an appropriate input data set is identified by carefully analyzing the various combinations of the groundwater levels \( H \) for three wells (as shown in Fig. 1) at various time lags, considering well 1 & 2 as observation wells and 3 as target well. Table I is giving the details of the input combinations considered in this study. \( H \) is referring to the ground water levels, \( T \) is the temperature and \( R \) is representing the amount of precipitation.

<table>
<thead>
<tr>
<th>Model Case</th>
<th>Input Combinations</th>
<th>Total Number of inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( H,(t), H,(t-1), H,(t), H,(t-1), H,(t), H,(t-1), R,(t), T,(t), T,(t-1) )</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>( H,(t), H,(t-1), R,(t), T,(t), T,(t-1) )</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>( H,(t), H,(t-1), H,(t), H,(t-1), H,(t), H,(t-1) )</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>( H,(t), H,(t-1), H,(t-2) )</td>
<td>3</td>
</tr>
</tbody>
</table>
Table II depicts the SVM parameters for the different models using QPSO as mentioned in section 3. Due to lack of any prior knowledge on the bounds of SVM parameters, a two-step QPSO search algorithm (12) is recommended. In the first step, a coarse range search is made to achieve the best region of the three-dimensional grids. Since doing a complete grid-search may still be time-consuming, a coarse grid search is recommended first. After identifying a better region on the grid, a finer grid search on that region can be conducted. In the present study, the coarse range partitions for C are taken as $[10^{-5}, 10^5]$. Similarly, the coarse range partitions for $\epsilon$ are taken to be $[0, 10^1]$ and the coarse range partitions for $\gamma$ are $[0, 10^1]$. Once the better region of grid is determined then a search is conducted in the finer range. Thus in the second stage search the parameter C ranges between $[10^{-1}, 10^1]$, is taken to be $[10^{-7}, 10^{-1}]$ and $\gamma$ is taken to $[0, 1]$. The program terminates when the RMSE value of the SVM model is less than $10^{-2}$ or it terminates after 1000 iterations. For real field cases it is difficult to obtain such low RMSE values so in general program terminates after 1000 iterations.

### TABLE III

**OPTIMAL SVM PARAMETERS OBTAINED FROM QPSO FOR DIFFERENT MODELS**

<table>
<thead>
<tr>
<th>Model</th>
<th>C</th>
<th>$\epsilon$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.78</td>
<td>0.101</td>
<td>0.975</td>
</tr>
<tr>
<td>2</td>
<td>1.056</td>
<td>0.017</td>
<td>0.6</td>
</tr>
<tr>
<td>3</td>
<td>2.117</td>
<td>0.076</td>
<td>0.471</td>
</tr>
<tr>
<td>4</td>
<td>1.692</td>
<td>0.111</td>
<td>0.935</td>
</tr>
</tbody>
</table>

As seen from the Table III, the SVM-QPSO models are evaluated based on their performance. The maximum coefficient of determination ($R^2$) obtained was 0.945 (in model 2) and the lowest $R^2$ term was 0.84 (model 4). In addition to this, model 2 exhibits the maximum value of efficiency (0.82), minimum MAPE (0.021) and minimum RMSE value (0.165). Model 2 which consists of two lags of groundwater level shows the highest efficiency, correlation, MAPE and minimum RMSE. As a result, model 2 has been selected as the best-fit model to estimate the groundwater level.

### TABLE III

**PERFORMANCE MEASURES FOR VARIOUS MODELS**

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE</th>
<th>MAPE</th>
<th>$R^2$</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.23</td>
<td>0.045</td>
<td>0.848</td>
<td>0.71</td>
</tr>
<tr>
<td>2</td>
<td>0.165</td>
<td>0.021</td>
<td>0.945</td>
<td>0.82</td>
</tr>
<tr>
<td>3</td>
<td>0.28</td>
<td>0.062</td>
<td>0.89</td>
<td>0.77</td>
</tr>
<tr>
<td>4</td>
<td>0.21</td>
<td>0.051</td>
<td>0.84</td>
<td>0.63</td>
</tr>
</tbody>
</table>

### V. COMPARISON WITH OTHER FORECASTING MODELS

The forecasting accuracy of the proposed SVM-QPSO model is compared with neural networks. ANN model developed in this study has architecture of 6-13-1 for the model 2. The ANN is developed for model 2 so that the results can be used for direct comparison. The architecture of ANN is obtained by a trial and error procedure. Table 4 is showing the performance of comparison of SVM model with ANN. For the testing data, SVM-QPSO model has RMSE of 0.17 whereas it is 0.296 for ANN. Thus SVM-QPSO shows an improved performance compared to other models. Similarly the correlation coefficient also improves when one moves from ANN to SVM-QPSO, as SVM-QPSO predicts the groundwater level values with a $R^2$ value of 0.94.

Further the results elucidates that even though SVM model is a very good regression tool but if the parameters of SVM are not trained properly then it performs very poorly, as in this case. Here the performance of SVM is poor compared to that of ANN. Whereas SVM-QPSO performs much better compared to ANN also because of the appropriate choice of parameters.
VI. SUMMARY AND CONCLUSIONS

Forecasting groundwater levels accurately is very useful for planning and management of water resources. Thus an attempt is made in this study to forecast groundwater levels in the Visakhapatnam region of Andhra Pradesh by developing a forecasting model using SVM-QPSO. The SVM-QPSO model is the combination of QPSO and SVM. SVM is based on structural minimization rather than the minimization of the errors, whereas QPSO chooses the optimal SVM parameters to improve the performance of the SVM model. Thus the integrating QPSO with SVM has made the proposed SVM-QPSO model to perform better compared to the other models. Many Performance measures like root mean square error (RMSE), Nash Sutcliffe efficiency and coefficient of correlation were used in this study for comparing the performance of SVM-QPSO with the other existing popular forecasting techniques. SVM-QPSO outperforms other forecasting models in all the cases. Thus SVM-QPSO model can be better alternative for forecasting groundwater level.

REFERENCES


